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Comparison of Lossy versus Lossless Shift Factors in the ISO Market Optimizations

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1. Introduction

The optimization algorithms used in the ISO markets calculate and utilize shift factors, also known as Power Transfer Distribution Factors (PTDFs), as an essential part of the process of determining resource schedules and prices. For any given pricing node (PNode) in the full network model (FNM), the shift factors associated with that PNode describe the resulting flows across all network lines[1] when one additional MW of energy is injected at the PNode and withdrawn at the specified slack bus or buses. There are two main types of shift factors that may be used. One type is known as "lossless" shift factors. These are calculated under the assumption of a lossless network; thus they describe how the additional one MW would flow over the FNM without any thermal losses, so that the entire one MW is then withdrawn at the slack. The other type is known as "lossy" shift factors. These are calculated taking into account the thermal losses that will affect the one MW injection, so that a quantity somewhat less than one MW is able to be withdrawn at the slack to keep the system balanced.

The new ISO markets have been using lossless shift factors since they started operation with the first running of the integrated forward market (IFM) on March 31, 2009. Indeed, lossless shift factors had been used throughout market simulation, and the use of lossless shift factors is specified in the ISO tariff (see Appendix C, sub-section c on the subject of PTDFs). Moreover, all other U.S. ISOs and RTOs use lossless shift factors.

Since start-up, however, the ISO has observed instances in which the dispatch software has resorted to relatively ineffective resource adjustments in attempting to relieve transmission constraints that could not be resolved in the scheduling run. In some instances, the cause for such ineffective adjustments could be traced to the fact that the dispatch software was using lossless shift factors to re-dispatch transmission constraints while taking full account of losses in solving the power balance equation. Said another way, there are certain types of constrained system conditions where the use of lossless shift factors causes the dispatch software to adjust resource schedules in ways that appear to be more effective in solving transmission constraints than they really are, and more effective than they would appear to be if lossy shift factors were used in the re-dispatch. Because these types of market conditions can have significant but spurious price impacts in those five-minute dispatch intervals when they do occur, the ISO is considering whether it would be beneficial to market performance to adopt the use of lossy shift factors in the market optimizations. The purpose of this technical bulletin is to provide an

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[1] Although in theory shift factors for any given PNode can be calculated with respect to all network lines, in the ISO's market applications shift factors are calculated only for transmission elements that have been determined to be on the critical constraint list for any given market interval. The critical constraint set includes all nomograms in all market intervals, plus any transmission elements for which the flow is greater than or equal to a threshold percent of the flow limit in the given market interval.

explanation and some examples to illustrate how these two types of shift factors work in the market software, and to identify some of the pros and cons of each type.

2. Technical discussion of lossy versus lossless shift factors

In a mathematical model of an electricity network, such as the Full Network Model (FNM) used in the ISO markets, each network node has an associated set of power transfer distribution factors (PTDFs) or shift factors. The shift factors for a particular node n describe how power will flow over the network when one MW is injected at node n and withdrawn at another location referred to as the slack or reference location. The slack location can be a single network node or a set of network nodes; the latter case is known as a distributed slack. (The ISO markets utilize a distributed load slack,[2] where the distribution of the slack is based on the same load distribution factors used to distribute the ISO load forecast for those market processes that utilize the load forecast.)

Thus the shift factors for a node n will depend on the FNM topology – the map of all the lines and their interconnections, and the electrical characteristics of the lines – and on the specification of the slack bus or buses. Given the topology and the slack, for any node n and transmission line j in the FNM, the shift factor SFnj is the amount of energy that will flow over line j in the pre-specified reference direction when 1 MW is injected at node n. Thus all shift factors are numbers between -1.0 and +1.0.

The shift factor or PTDF expresses the amount of active power that flows on a given network branch in a given direction for a marginal active power injection at a given network node. A positive shift factor indicates that a marginal active power injection at the node increases the active power flow on the branch and direction, whereas a negative shift factor indicates that a marginal active power injection at the node decreases the active power flow on that branch and direction. The concept can be generalized to transmission interfaces (branch groups) by adding the shift factors of each branch in the group, and also to nomograms that are linear combinations of transmission interface flows by adding the shift factors of each transmission interface in the nomogram multiplied by the relevant coefficients. The shift factors are always calculated with respect to a reference or slack bus. The reference is the location that absorbs the active power that is injected to maintain the active power balance in the network. By definition, all shift factors at the reference are zero since all active power is injected and absorbed there without flowing on any branch.[3]

The shift factors are used in power system optimization and market clearing applications to dispatch resources to solve network constraints. In these applications, the change in the

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2 Until April 18, IFM utilized a distributed generation slack reference.

3 In the case of a distributed reference or slack over multiple buses, the last sentence requires thinking of the slack as a single aggregate resource formed as a weighted sum of the constituent buses, so that a one-MW injection distributed to the constituent buses according the specified weights is also withdrawn at each of the constituent buses according to the same weights, so no power flows over the network.

active power flow over a network branch from an AC power flow solution is approximated linearly as the sum of the changes in the active power injections multiplied by the relevant shift factors. The error from this linearization is corrected and reduced iteratively by iterating between the optimization solver (i.e., the security constrained unit commitment or SCUC algorithm, which determines resource schedules) and an AC power flow. A network constraint that is binding at the optimal solution contributes to the marginal congestion price at a node an amount equal to the negative product of the relevant constraint shadow price and the shift factor on that constraint.

Traditionally, shift factors are calculated ignoring losses between a network node and the reference, i.e., the reference absorbs exactly the same amount of active power with that injected at the node. Lossless shift factors offer simplicity and computational advantages. Because they do not depend on losses, they are linear, allowing superposition. Because of superposition, the congestion cost between any two nodes in the system, i.e., the difference between the marginal congestion prices at these nodes, is independent of the reference used in the calculation. Additionally, lossless shift factors depend only on network configuration and the slack designation, and not on the level or pattern of power flows on the network. Therefore, lossless shift factors remain constant between iterations in market clearing applications, and even across markets (such as between day-ahead and real-time) unless there are network changes, e.g., transmission outages or switching, or changes to the designation of the slack.

Aside from these advantages, lossless shift factors also have some shortcomings. The shift factor at a node with respect to a branch is a measure of the effectiveness of a power injection at that node in mitigating congestion at that branch. The absence of losses in the shift factor calculation removes the natural attenuation that losses cause in that effectiveness. As a result, a node may appear to the optimization to retain significant effectiveness even when it is electrically remote from the branch. A relevant example for the ISO markets is radial congestion where all nodes on the same side of the congested interface have the same lossless shift factor impact on the constraint irrespective of their electrical distance to that constraint. In other words, using lossless shift factors the optimization will see a generating unit in the Bay Area and a dynamic import at PACI as equally effective in mitigating Path 26 congestion.

From an optimization perspective, when a network constraint cannot be resolved and its violation triggers a high penalty price in the objective function, lossless shift factors may result in some relatively inefficient dispatch among resources where large amounts of re-dispatch provide very small congestion relief. For example, the optimization may re-dispatch two resources on the same side of a radial constraint, one in the upward direction and another in the downward direction in nearly equal quantities, because differences in the loss impacts of these two resources allows a slight reduction in the total generation needed to meet load plus losses. Under lossless shift factors, this slight reduction in total generation results in a slight reduction in flow across the overloaded radial constraint. This re-dispatch comes at a high cost, but still lower than the penalty price, resulting in high constraint shadow prices and sometimes high LMPs at some locations.[4] These high

[4] Such an event occurred on 4/19/2009 resulting in a high DLAP price for SDG&E for 15:00-15:05. The details are explained in the ISO Technical Bulletin 2009-05-02.

prices do not really provide meaningful economic signals, however, in circumstances in which the difference in shift factors between the resources dispatched to solve the constraint is very small relative to rounding conventions and approximations in the dispatch model.[5]

3. Illustrative examples

This section provides simple three-bus examples to illustrate how the choice of lossy or lossless shift factors can also affect market prices and net market revenues. In particular, using lossless rather than lossy shift factors can result in negative net congestion revenues for a market interval (example 1) and negative net energy revenues (example 2). The scenarios in these examples are too simple to illustrate the ineffective re-dispatch effect discussed in the previous section; for an example of that effect the reader is referred to the technical bulletin mentioned above on the April 19 prices.

Example 1. The first example illustrates how the use of lossless shift factors can cause net total congestion revenue for a market interval to be negative. Total net congestion revenue for a market interval is defined as the sum over all pricing locations of the marginal congestion cost (MCC) component of the applicable LMP times the net energy injection at that location, plus any congestion charges to ancillary services procured on the interties. Specifically:

Total net congestion revenue =

Sum over all load locations (MCC * MWh of load)

+ Sum over all export locations (MCC * MWh of export)

- Sum over all generator locations (MCC * MWh of generation)

- Sum over all import locations (MCC * MWh of import)

+ Sum over all import locations (shadow price of intertie constraint * MW-hours of procured AS).

The following scenario demonstrates how this quantity can be negative as a result of using lossless shift factors, and becomes positive using lossy shift factors.

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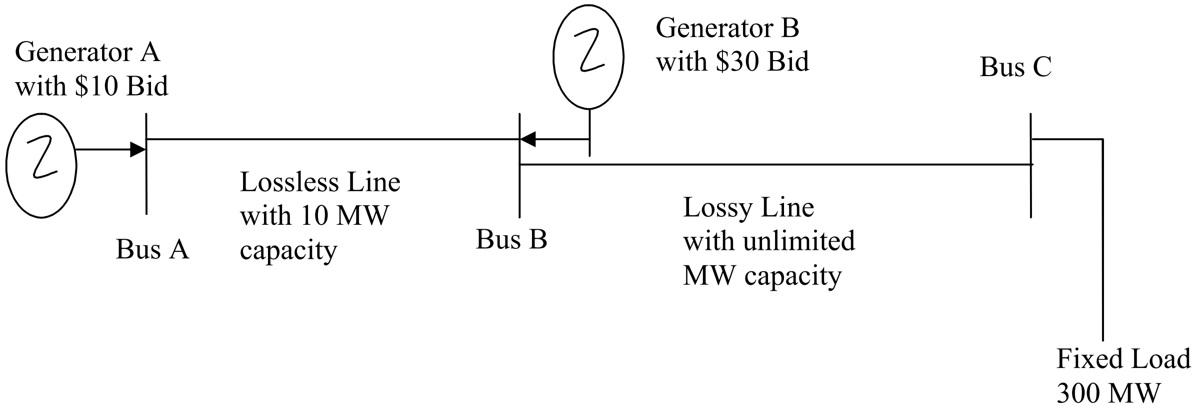
5 At first glance it might seem like the ISO's two percent effectiveness threshold should prevent the use of such ineffective re-dispatches. As discussed in the ISO's March 23, 2009 compliance filing on market parameters (http://www.caiso.com/237a/237ae76b46410.pdf) the effectiveness threshold would eliminate individual resources from being re-dispatched to resolve a constraint on which they are less effective than the threshold. The situation described above could still occur, however, in the event that both resources are individually more effective than the threshold, but the combination of decrementing one resource and incrementing the other is very ineffective. The threshold filters out only individual resources of low effectiveness, and cannot filter out re-dispatch solutions involving multiple resources where the total re-dispatch has extremely low effectiveness.



Since generator A is relatively inexpensive, it is dispatched at 10MW and the lossless line AB becomes binding at 10MW flow. The more expensive generator B is dispatched for the remaining energy needed to meet the fixed 300 MW load and the transmission losses along the lossy line BC. For delivering 300MW at bus C assume that the transmission losses are 15MW, corresponding to 5 percent average loss. Therefore, generator B is dispatched at 305MW. With 5 percent average loss for the lossy transmission line, the marginal loss is about 10 percent, i.e. 2 times the average loss percentage. This means that with one MW additional injection at B, we can only withdraw 0.9MW at the load location C due to incremental transmission losses.

Since line AB is binding, its shadow price is $30 - $10 = $20. For future reference, we calculate the MW flow times the shadow price, which is equal to $200.

We next calculate the locational marginal price decomposition with bus A chosen as slack, using both lossless and lossy shift factors. (Although this example uses a generator bus as the slack, the salient difference between lossless and lossy shift factors that this example demonstrates will hold true under a load slack as well.)

Using Lossless Shift Factors

The LMPs at bus A and B are respectively $10 and $30. With bus A as slack, the loss penalty factor of B is 1 and of C is 0.9.[6] The loss sensitivity of each bus satisfies the equation Loss Penalty Factor = 1/(1-Loss Sensitivity). In this example the loss sensitivity is 0 for bus B and (1 – 1/0.9) = -0.111 for bus C.

Let the reference direction of line AB be from A to B. With respect to this line and the designation of A as the slack bus, lossless shift factors of bus B and C are both -1.

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[6] Loss penalty factors are used to account for the impact on system losses due to a change in supply or demand at any network location. The loss penalty factor for a bus N is defined as the amount of energy that must be injected (or withdrawn) at N in order to withdraw (or inject) one MW at the slack bus. The value of the loss penalty factor depends on the direction of flow on the network. In this example, because the underlying flow is towards the load at bus C, one MW withdrawn at slack bus A served by an injection at C would actually reduce system losses. Therefore only 0.9 MW injection (load reduction) at C is needed to balance one MW withdrawal at A.

Next we calculate the LMP of each bus to reveal the decomposition into system energy, congestion and loss components. The following formula is used for the calculation:

LMP of a bus = LMP of slack bus – (LMP of slack * loss sensitivity of bus)

– (shift factor of bus * shadow price of line AB).

The first term of the right hand side is the energy component, the second term is the loss component and the third term is the congestion component. The LMPs and their decomposition for all three buses are given in the table below.

	LMP Energy Component	LMP Loss Component	LMP Congestion Component	LMP
Bus A (slack)	$10	$0	$0	$10
Bus B	$10	$0	$20	$30
Bus C	$10	$1.111	$20	$31.111

Because generator B and load C have the same LMP congestion component at $20, and the MW of generator B is 5MW above the load C, the net congestion revenue for this market interval is $20 * 300 MWh - $20 * 305 MWh = -$100. This demonstrates that congestion revenue could become negative under lossless shift factor. As stated in the introduction, this result occurs because the optimization is using lossless shift factors to deal with congestion and calculate the congestion components of the LMPs, while taking full account of losses in maintaining energy balance for the system. Note in particular that the shift factors used in the LMP equation are the lossless shift factors, while the MW dispatches from A and B are based on generating enough energy to balance load plus losses.

<u>Using Lossy Shift Factors</u>

With bus A as slack, the lossy shift factor of bus C is –1/0.9 = -1.111 because injecting one MW into bus C will reduce the flow on the congested line from A to B by 1.111 MW. The lossy shift factor of bus B is –1, equal to the lossless shift factor because AB was assumed to be a lossless line. The LMPs and their decomposition for all 3 buses are given in the table below.

	LMP Energy Component	LMP Loss Component	LMP Congestion Component	LMP
Bus A (slack)	$10	$0	$0	$10
Bus B	$10	$0	$20	$30
Bus C	$10	$1.111	$22.222	$33.333

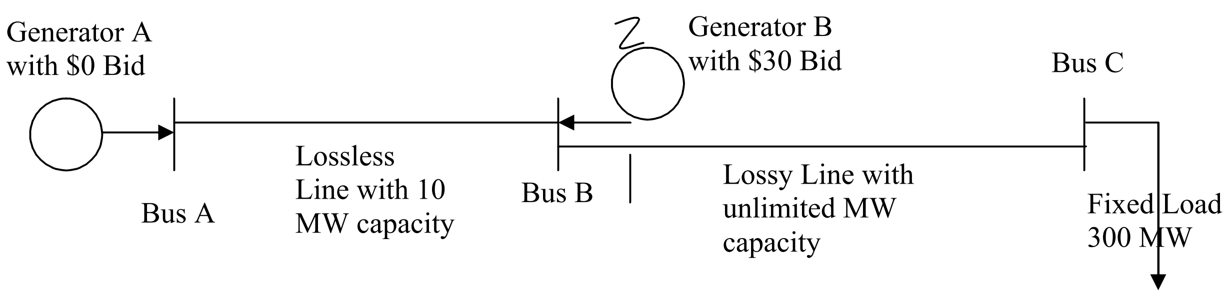
Congestion revenue = $22.222 * 300 MW – $20 * 305 MW = $566.67 is now positive.

This example demonstrates the principle that with marginal transmission losses approximately equal to 2 times average losses and with the use of lossy shift factors in the SCUC optimization formulation, the pricing outcome of the market solution will guarantee positive net congestion revenue under flowgate congestion. This is also true regardless of the choice of slack bus.

As a variation to the example above, we could choose the load bus C as slack. In this case the LMP congestion components of B and C are both zero and congestion component of A is −$20. As a result the net congestion revenue is $200. The same principle does not apply with the use of lossless shift factors, however. With lossless shift factors the net congestion revenues may be positive or negative depending on the specifics of the problem and the choice of slack.

Example 2. The second example shows how the total net energy revenue for a market interval can be negative as a result of using lossless shift factors. Total net energy revenue for a market interval is defined as the sum over all demand locations of (LMP * MWh of demand) minus the sum over all supply locations of (LMP * MWh of supply).

The example is structured similarly to the previous example. The only change being made is that the bid price of generator A is reduced from $10 to $0.



The MW solution is the same as example 1 with Generator A dispatched at 10MW and Generator B dispatched at 305MW.

Since line AB is binding, its shadow price is $30 - $0 = $30.

We next calculate the locational marginal prices and their decomposition with bus A chosen as slack.

The LMPs at bus A and B are respectively $0 and $30. With bus A as slack, the loss penalty factor of B is 1 and of C is 0.9 as in the previous example, and the loss sensitivity is 0 for bus B and (1 – 1/0.9) = -0.111 for bus C.

Let the reference direction of line AB be from A to B. With respect to this line, lossless shift factors of bus B and C are both –1.

Using lossless shift factors the LMPs and their decomposition for all 3 buses are given in the table below.

	LMP Energy Component	LMP Loss Component	LMP Congestion Component	LMP
Bus A (slack)	$0	$0	$0	$0
Bus B	$0	$0	$30	$30
Bus C	$0	$0	$30	$30

Note that since LMP of the bus A (slack) is $0, loss components of all 3 buses are zero.

Since generator B and load C have the same LMP at $30 and MW of generator B is 5MW above the load C, total revenue is $30 * 300 MWh - $30 * 305 MWh = -$150. This demonstrates that total revenue could become negative under lossless shift factor.

With lossy shift factors the LMP at bus C would be $33.333 and the total revenue total would be positive, i.e., $33.33 * 300 MWh - $30 * 300 MWh = $999.99.