Comments on Demand Curves in the Flexible Ramping Product Draft Technical Appendix

Department of Market Monitoring

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1 Summary

Flexible ramping product procured in one interval provides the option to respond to the uncertainty in the net load of future intervals. The value of flexible ramping product in a real-time interval is the amount that expected costs in subsequent real-time intervals change because this flexible capacity is available to respond to net load outcomes that may differ from the load forecast. The demand curve for flexible ramping product should represent the reduction in expected power balance constraint violation costs due to procuring additional amounts of flexible capacity.

The ISO has previously adopted the general framework initially proposed by the Department of Market Monitoring (DMM) for using the distribution of net load forecast errors to calculate the demand curve. We support this general framework.[1] In the *Flexible Ramping Product Draft Technical Appendix*, the ISO is now specifying more precisely how it plans to implement this general framework. In these comments we propose an alternative to the ISO's specific demand curve formulation. We believe this alternative formulation more accurately reflects the value of flexible ramping products and is relatively easy to calculate.

Equation 1 shows DMM's proposed demand curve for Flexible Ramping Up (FRU) capacity. The marginal value of FRU is the probability that the net load forecast error is greater than or equal to the quantity of upward flexible capacity multiplied by the Power Balance Constraint shortage penalty price.

Equation 2 shows DMM's proposed demand curve for Flexible Ramping Down (FRD) capacity. The marginal value of FRD is the probability that the forecast error is less than or equal to the quantity of downward flexible capacity (where downwards capacity is measured in negative megawatts) multiplied by the Power Balance Constraint excess generation penalty price.[2]

Equation 1. Upward Flexible Ramping Capacity Demand Curve

$$Price(FRU) = PC^{short} * \sum_i p_i \quad \forall\, p_i \text{ } of \text{ } \varepsilon_i \geq FRU$$

Equation 2. Downward Flexible Ramping Capacity Demand Curve

$$Price(FRD) = PC^{excess} * \sum_i p_i \quad \forall\, p_i \text{ } of \text{ } \varepsilon_i \leq FRD$$

[1] See Appendix A of DMM's comments on the Flexible Ramping Products Incorporating FMM and EIM Straw Proposal, July 7, 2014, available at: http://www.caiso.com/Documents/DMM-CommentsFlexibleRampingProductsStrawProposal.pdf

[2] If Flexible Ramping Down capacity is shown as a positive value then this needs to be multiplied by -1.

2 Examples of Calculating Flexible Ramping Product Demand Curves

Table 1 shows a hypothetical distribution of net load forecast errors represented by a histogram. From this histogram upward and downward flexible capacity demand curves can be derived using the formulas in Equation 1 and Equation 2. Prices are calculated at the mid-point of each histogram segment as this will be the average price within the segment.[3]

Table 1. Example Probability Distribution and Flexible Capacity Demand Curve

Beg MW	End MW	Pr Density	PBC Penalty	FRP Price*	
-400	-350	0.25%	($155)	**($0.19)**	FRD Demand Curve
-350	-300	0.50%	($155)	**($0.78)**	
-300	-250	1.25%	($155)	**($2.13)**	
-250	-200	2.50%	($155)	**($5.04)**	
-200	-150	5.50%	($155)	**($11.24)**	
-150	-100	8.00%	($155)	**($21.70)**	
-100	-50	12.00%	($155)	**($37.20)**	
-50	0	20.00%	($155)	**($62.00)**	
0	50	20.00%	$1,000	**$400.00**	FRU Demand Curve
50	100	12.00%	$1,000	**$240.00**	
100	150	8.00%	$1,000	**$140.00**	
150	200	5.50%	$1,000	**$72.50**	
200	250	2.50%	$1,000	**$32.50**	
250	300	1.25%	$1,000	**$13.75**	
300	350	0.50%	$1,000	**$5.00**	
350	400	0.25%	$1,000	**$1.25**	

*At mid-point

The marginal value of upward flexible capacity at 225 MW with a $1,000 penalty price is:

$$Price(y_j = 225) = 1{,}000 * (0.025/2 + .0125 + 0.005 + 0.0025) = 1{,}000 * 0.045 = \$32.50$$

The marginal value of downward flexible capacity at -150 MW with a -$155 penalty price is:

$$Price(y_k = -175) = -155 * (0.055/2 + 0.025 + 0.0125 + 0.005 + 0.0025) = -155 * 0.0725 = -\$11.24$$

[3] The price at the mid-point of a histogram segment is the average price within the segment assuming each error within the segment is equally likely to occur as any other error within the segment.

3 Derivation of Flexible Ramping Product Demand Curves

The market optimization minimizes total production costs. These include the costs of violating the power balance constraint when there is either insufficient or excess generation. The power balance constraint violation costs are equal to the shortage or excess megawatt amount multiplied by the associated penalty price.

Equation 3 below shows a two period optimization where the second period net load is uncertain. In the first period the shortage or excess megawatts, R_1^{short} or R_1^{excess}, are known and the costs can be calculated from these amounts. In the second period the shortage and excess megawatts are not known and their cost functions are instead represented by the expected costs given the amount of flexible capacity available, Y_t, and the distribution of net load forecast errors, ε_2. The maximum and minimum operators are included as having more available upward or downward flexible capacity then net load changes does create negative power balance constraint violation costs. A list of the notation used is provided at the end of these comments.

Equation 3. Objective Function with Uncertain Net Load in Second Period

$$\min_{MW_i, Y_1^{up}, Y_1^{down}} c\left(MW_{i,t}, Y_1^{up}, Y_1^{down}\right) = \sum_{t=1}^{2} \sum_{i} MW_{i,t} * c\left(MW_{i,t}\right) + R_1^{short} * PC^{short} + R_1^{excess} * PC^{excess}$$
$$+ \int_{-\infty}^{+\infty} PC^{short} * \max\left(0, \varepsilon_2 - Y_1^{up}\right) P(\varepsilon_2) d\, \varepsilon_2$$
$$+ \int_{-\infty}^{+\infty} PC^{excess} * \min\left(0, \varepsilon_2 - Y_1^{down}\right) P(\varepsilon_2) d\, \varepsilon_2$$

Equation 4. Power Balance Constraints

$$\sum_{i} MW_{i,1} + R_1^{short} + R_1^{excess} = 0$$

$$\sum_{i} MW_{i,2} + R_2^{short} + R_2^{excess} = 0$$

The errors, ε_2, are the differences between the realized net load and the expected net load. The objective function is defined here in terms of the errors as in practice it will likely be easier to estimate a distribution of errors rather than net loads as an input into the optimization. $Y_1^{up/down}$ is the available flexible capacity up or down from the first interval to the next, less the expected ramp, i.e. centered on the expected net load just as the errors are. Note that Y_1^{up} can take negative value even though $\sum_i FRU_i$ is positive and Y_1^{down} can take positive value even though $\sum_i FRD_i$ is negative. This is done to align the choice variable with the errors and makes it more convenient to take derivatives.

Equation 5. Re-Aligning Flexible Capacity Choice Variable with Expected Net Load

$$Y_{t=1}^{up} = \sum_i FRU_i - (E(NL_2) - NL_1)$$

$$Y_{t=1}^{down} = \sum_i FRD_i - (E(NL_2) - NL_1)$$

An optimization could be constructed as in Equation 3 to be solved directly. The optimal solution would find where the marginal value of reduced expected power balance violation costs in period two equal the marginal cost of procuring flexible capacity on the rest of the optimization.

However the ISO is not planning on including the expected power balance costs directly in the optimization as this would likely be cumbersome and require greater changes to the software than the proposed inclusion of a constraint. The marginal value of the decreased expected power balance constraint violations due to increasing flexible capacity needs to be found to create a demand curve that defines how the flexible ramping product constraint is relaxed as the costs of procuring capacity increase. In this way the optimization can make the same trade off in costs to procure flexible capacity versus the value of different levels of total flexible capacity that is made in the optimization shown in Equation 3.

Taking the derivative of the expected power balance constraint violation costs defines how these costs change as more flexible capacity is procured. The expected shortage and excess costs in the objective function can both be split into two at the point where $Y_{t=1}^{up/down} = \varepsilon$. This way one part of the shortage/excess cost function is zero while the other part is differentiable.

Equation 6. Splitting Out Expected Power Balance Constraint Costs

$$E[C(short)] = \int_{Y_1^{up}}^{+\infty} PC^{short} * (\varepsilon_2 - Y_1^{up}) * P(\varepsilon_2) d\,\varepsilon_2 + \int_{-\infty}^{Y_1^{up}} PC^{short} * 0 * P(\varepsilon_2) d\,\varepsilon_2$$

$$E[C(excess)] = \int_{-\infty}^{Y_1^{down}} PC^{excess} * (\varepsilon_2 - Y_1^{down}) * P(\varepsilon_2) d\,\varepsilon_2 + \int_{Y_1^{down}}^{+\infty} PC^{excess} * 0 * P(\varepsilon_{j,2}) d\,\varepsilon_2$$

The derivative of the Equation 6 cost functions are the change in expected power balance costs due to increased flexible capacity. Equation 7 shows the steps to find the derivative of the expected power balance constraint shortage violations costs. A similar approach can be used for the down product. Step 1 is expanding the non-zero portion of the cost function into two parts. Step 2 applies the fundamental theorem of calculus to find the derivative of the first part of the expanded function. Step 3 applies the fundamental theorem of calculus and the chain rule to find the derivative of the second part. Step 4 distributes the negative sign throughout the second part. Step 5 reduces the equation as the first and last terms in Step 4 cancel out.

Equation 7. Derivative of Expected Power Balance Constraint Costs - Flexible Ramping Up

$$\frac{\partial E[C(short)]}{\partial Y_1^{up}} =$$

$$step1 = \frac{\partial}{\partial Y_1^{up}} \int_{Y_1^{up}}^{+\infty} PC^{short} * \varepsilon_2 * P(\varepsilon_2) d\,\varepsilon_2 - \frac{\partial}{\partial Y_1^{up}} \int_{Y_1^{up}}^{+\infty} PC^{short} * Y_1^{up} * P(\varepsilon_2) d\,\varepsilon_2$$

$$step2 = -PC^{short} * Y_1^{up} * p_t\left(Y_1^{up}\right) - \frac{\partial}{\partial Y_1^{up}} \int_{Y_1^{up}}^{+\infty} PC^{short} * Y_1^{up} * P(\varepsilon_2) d\,\varepsilon_2$$

$$step3 = -PC^{short} * Y_1^{up} * p_t\left(Y_1^{up}\right) - \left[\int_{Y_1^{up}}^{+\infty} PC^{short} * P(\varepsilon_2) d\,\varepsilon_2 - PC^{short} * Y_1^{up} * p_t\left(Y_1^{up}\right)\right]$$

$$step4 = -PC^{short} * Y_1^{up} * p_t\left(Y_1^{up}\right) - \int_{Y_1^{up}}^{+\infty} PC^{short} * P(\varepsilon_2) d\,\varepsilon_2 + PC^{short} * Y_1^{up} * p_t\left(Y_1^{up}\right)$$

$$step5 = -\int_{Y_1^{up}}^{+\infty} PC^{short} * P(\varepsilon_2) d\,\varepsilon_2$$

The derivative of the expected cost function multiplied by negative one is the marginal value of the additional flexible capacity, i.e. how much the optimization would be willing to spend to procure the capacity. Equation 8 and Equation 9 show the demand curves for upward and downward flexible ramping capacity.

Equation 8. Demand Curve for Flexible Ramping Up Capacity

$$-1 * \frac{\partial E[C(short)]}{\partial Y_1^{up}} = \int_{Y_1^{up}}^{+\infty} PC^{short} * P(\varepsilon_2) d\,\varepsilon_2$$

Equation 9. Demand Curve for Flexible Ramping Down Capacity

$$-1 * \frac{\partial E[C(excess)]}{\partial Y_1^{down}} = \int_{-\infty}^{Y_1^{down}} PC^{excess} * P(\varepsilon_2) d\,\varepsilon_2$$

4 Demand Curve Derivation from Discrete Probabilities

Here the flexible ramping product demand curves are derived from discrete expected cost functions and an example of how the expected costs behave is shown to help clarify demand curve derivation.

For brevity the derivation for the flexible ramping up demand curve starts with the expected power balance constraint violation costs. Recall that the difference between the net load and the available flexible ramping up capacity is the power balance constraint shortage. The expected cost is the probability weighted sum of the shortages multiplied by the penalty price, Equation 10. Negative one multiplied by the derivative of the expected costs is the demand curve for flexible ramping up capacity, Equation 11. A similar operation is shown for the flexible ramping down product in Equation 12 and Equation 13.

Equation 10. Total Expected PBC (Shortage) Violation Cost Given Available FRU Capacity

$$E\left(C\left(\varepsilon_j, Y_1\right)\right) = max(0, \varepsilon_1 - Y_1) * PC^{short} * p_1 + max(0, \varepsilon_2 - Y_1) * PC^{short} * p_2 \ldots.$$

$$E\left(C\left(\varepsilon_j, Y_1\right)\right) = PC^{short} * \sum_j max\left(0, \varepsilon_j - Y_1\right) * p_i$$

Equation 11. Marginal Value of FRU Capacity

$$-1 * \frac{\partial E\left(C\left(\varepsilon_j, Y_1\right)\right)}{\partial Y_1} = PC^{short} * \sum_j p_j \quad \forall\, p_j\ of\ \varepsilon_j \geq Y_1$$

Equation 12. Total Expected PBC (Excess) Violation Cost Given Available FRD Capacity

$$E\left(C(x_i, y_k)\right) = r_k * \sum_i min(0, x_i - y_k) * p_i$$

Equation 13. Marginal Change in Expected Cost from FRD Capacity

$$\frac{\partial E\left(C\left(\varepsilon_i, y_j\right)\right)}{\partial y_j} = r_k * \sum_i -1 * p_i \quad \forall\, p_i\ of\ \varepsilon_i \leq y_k \quad + 0 * p_i \quad \forall\, p_i\ of\ \varepsilon_i > y_k$$

Table 2 shows an example of how total expected power balance constraint violation costs change as the quantity of available flexible ramping up capacity changes. When there is no flexible ramping up capacity the cost of a one megawatt error, if it were to occur, is $1,000. If a two megawatt error were to occur, the cost would be $2,000, and so on. The total expected cost is the sum of costs of the potential outcomes weighted by the probability that the outcome occurs. With no available flexible capacity the expected cost is $2,503.[4]

When one megawatt of FRU capacity is made available it changes the costs incurred due to various outcomes of the forecast error. Now a one megawatt error incurs no cost as the FRU capacity is deployed to avoid a PBC violation. A two megawatt error now only costs $1,000 as the there is only a one megawatt shortage after the available megawatt of FRU capacity is deployed. A three megawatt forecast error costs $2,000 and so on. The total expected costs given one megawatt of FRU capacity is $2,003. The first megawatt of FRU capacity reduced expected costs by $500.

Table 2. Example Error Distribution and PBC Violation Costs at Various Quantities of FRU Capacity

	Pr(Error=X)	6.25%	6.00%	5.75%	5.50%	5.25%	5.00%	4.50%	4.25%	4.00%	3.50%	Total Expected Cost	Change in Expected Cost
	Error	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**		
Available Flexible Capacity	**0**	$1,000	$2,000	$3,000	$4,000	$5,000	$6,000	$7,000	$8,000	$9,000	$10,000	$2,503	
	1	$0	$1,000	$2,000	$3,000	$4,000	$5,000	$6,000	$7,000	$8,000	$9,000	$2,003	**$500.00**
	2	$0	$0	$1,000	$2,000	$3,000	$4,000	$5,000	$6,000	$7,000	$8,000	$1,565	**$437.50**
	3	$0	$0	$0	$1,000	$2,000	$3,000	$4,000	$5,000	$6,000	$7,000	$1,188	**$377.50**
	4	$0	$0	$0	$0	$1,000	$2,000	$3,000	$4,000	$5,000	$6,000	$868	**$320.00**
	5	$0	$0	$0	$0	$0	$1,000	$2,000	$3,000	$4,000	$5,000	$603	**$265.00**
	6	$0	$0	$0	$0	$0	$0	$1,000	$2,000	$3,000	$4,000	$390	**$212.50**
	7	$0	$0	$0	$0	$0	$0	$0	$1,000	$2,000	$3,000	$228	**$162.50**
	8	$0	$0	$0	$0	$0	$0	$0	$0	$1,000	$2,000	$110	**$117.50**
	9	$0	$0	$0	$0	$0	$0	$0	$0	$0	$1,000	$35	**$75.00**
	10	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0	**$35.00**
	Pr(E≥X)	50.00%	43.75%	37.75%	32.00%	26.50%	21.25%	16.25%	11.75%	7.50%	3.50%		
	Pr(E≥X)*1,000	**$500.00**	**$437.50**	**$377.50**	**$320.00**	**$265.00**	**$212.50**	**$162.50**	**$117.50**	**$75.00**	**$35.00**		

At the bottom of Table 2 the total probability of forecast errors greater than or equal to a particular error is summed. Multiplying this by the $1,000 penalty price, as in Equation 11, yields the same marginal values to capacity as found when calculating the change in total expected costs.

[4] Note that there are no PBC violation costs due to shortages for forecast errors in the negative direction which may cause excess generation. The downward flexible ramp product is procured for these errors.

5 Comparison to Draft Technical Appendix Demand Curves

The ISO's Draft Technical Appendix uses the cost function shown below in Equation 14 to describe how the flexible ramping up demand curve will be formulated. This is defined in terms of the constraint surplus variables, i.e. the amount the constraints will be relaxed, rather than the amount of flexible ramping capacity. This may be less intuitive to some than looking from the point of view of the amount of flexible capacity, but is consistent with how the demand curve will be implemented as a series of surplus variables that start to relieve the constraint as the cost of procuring capacity increases.

The derivative of this cost function, Equation 15, is the demand function implied from the cost function in the Draft Technical Appendix. This is different than the demand curve we derived above. The difference between the ISO's specification of the demand curve and DMM's specification of the demand curve appears to be due to the cost function in the ISO's Draft Technical Appendix not being based on the expected power balance violation costs. In Equation 16, we alter our cost function to represent the expected power balance constraint defined in terms of the surplus variable. This puts DMM's specification of the cost function into similar notation as the ISO's specification. Therefore, the difference between the ISO's specification and DMM's specification of the cost function can be seen by contrasting Equation 14 with Equation 16.

Using similar steps as in Equation 7 the derivative of DMM's alternative cost formulation is shown in Equation 17. Equation 17 is now the same demand curve we defined above in Equation 8, but with the ISO's notation. Similarly our proposed alternative flexible ramping down cost function and demand curve are shown in Equation 18 and Equation 19, defined in terms of the surplus variable.

Equation 14. ISO FRU Cost Function in Draft Technical Appendix

$$CSU_t(FRUS_t) = PC * \int_{EU_t - FRUS_t}^{EU_t} e * p_t(e)de$$

Equation 15. ISO FRU Demand Curve Implied from Draft Technical Appendix

$$\frac{\delta CSU_t(FRUS_t)}{\delta FRUS_t} = PC * (EU_t - FRUS_t) * p_t(EU_t - FRUS_t)$$

Equation 16. DMM Alternative PBC Shortage Expected Cost Function Defined in Terms of Surplus Variable

$$CSU_t(FRUS_t) = PC * \int_{EU_t - FRUS_t}^{EU_t} \big(e - (EU_t - FRUS_t)\big) * p_t(e)de$$

Equation 17. DMM FRU Demand Curve Derived from Alternative Expected Cost Formulation

$$\frac{\delta CSU_t(FRUS_t)}{\delta FRUS_t} = PC * \int_{EU_t - FRUS_t}^{EU_t} p_t(e)de$$

Equation 18. DMM Alternative PBC Excess Expected Cost Function Defined in Terms of Surplus Variable

$$CSD_t(FRDS_t) = PF * \int_{ED_t}^{ED_t - FRDS_t} \big(e - (EU_t - FRDS_t)\big) * p_t(e)de$$

Equation 19. DMM FRD Demand Curve Derived from Alternative Expected Cost Formulation

$$\frac{\delta CSU_t(FRDS_t)}{\delta FRDS_t} = PF * \int_{ED_t}^{ED_t - FRDS_t} p_t(e)de$$

The ISO does not write out the derivative of its cost function in the Technical Appendix. Therefore, the ISO does not complete the specification of the demand curve in its Technical Appendix. In Equation 15, we wrote out the derivative of the cost function from the ISO's Technical Appendix in order to specify the demand curve that the ISO's cost function implies. The ISO's implied demand curve differs from the unspecified demand curve that the ISO uses to create the example in its Technical Appendix. Furthermore, both of these ISO demand curves differ from the demand curve that we propose.

In Table 3 we compare the demand curve(s) proposed by the ISO to DMM's proposed demand curve by constructing demand curves from the histogram used in the ISO's Technical Appendix.[5] In Table 4 we compare the three demand curve proposals using the histogram from our example above.

Table 3. Comparison of FRU Demand Curves Calculations Using Technical Appendix Histogram

Histogram Bin		Probability	Penalty	Demand Curve Alternatives		
				ISO	ISO	DMM
Start	End	Density	Cost	Example	Implied	Alternate
0	100	50.00%	$1,000	$500.00	$253.00	$272.00
100	200	1.40%	$1,000	$14.00	$21.00	$15.00
200	300	0.50%	$1,000	$5.00	$13.00	$5.50
300	400	0.30%	$1,000	$3.00	$11.00	$1.50

[5] The values for the ISO's "implied" demand curve are the average of the derivatives of the ISO's Technical Appendix cost functions evaluated at individual megawatt levels within the bin.

Table 4. Comparison of FRU Demand Curves Calculations Using another Example Histogram

Histogram Bin		Probability	Penalty	Demand Curve Alternatives		
				ISO	ISO	DMM
Start	End	Density	Cost	Example	Implied	Alternate
0	50	20.00%	$1,000	$120.00	$102.00	$400.00
50	100	12.00%	$1,000	$65.00	$181.00	$240.00
100	150	8.00%	$1,000	$55.00	$201.00	$140.00
150	200	5.50%	$1,000	$42.50	$193.00	$72.50
200	250	2.50%	$1,000	$20.00	$113.00	$32.50
250	300	1.25%	$1,000	$10.00	$69.00	$13.75
300	350	0.50%	$1,000	$5.00	$33.00	$5.00
350	400	0.25%	$1,000	$2.50	$19.00	$1.25

6 Notation Description

$FRU_i = flexible\ ramping\ up\ product\ award\ from\ resource\ i$

$FRD_i = flexible\ ramping\ down\ product\ award\ from\ resource\ i$

$MW_i = energy\ generation\ or\ load\ schedule\ for\ resource\ i$

$Y_t = total\ flexible\ ramping\ product\ less\ net\ load\ forecast$

$\varepsilon = net\ load\ forecast\ error$

$PC = penalty\ cost\ to\ relax\ constraint$

$R = constraint\ relaxation\ parameter$

$NL = net\ load$

$i = indexes\ individual\ resources$

$t = indexes\ trade\ intervals$