Locational Marginal Pricing (LMP): Basics of Nodal Price Calculation

CRR Educational Class #2

CAISO Market Operations
Why are LMPs important to the CRR Allocation & Settlement Process

- The CRR revenue stream calculation uses the LMPs that are calculated in the Day-ahead Integrated Forward Market.
Course Objectives

- Upon completion of this course, you will be able to:
  - Understand the basic definition of an LMP
  - Understand the important characteristics of LMPs
Agenda

- LMP Overview
- LMP Definitions
- LMP Characteristics
- LMP Settlements
- LMP Observations
- BREAK
- LMP Examples (unconstrained cases)
- LMP Examples (constrained cases)
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Simplifying assumption for this presentation

The Integrated Forward Market
- Co-optimizes both energy and ancillary services (thus the use of the term “Integrated”)
- Incorporates a Full Network Model
- Clears congestion with no distinction between the currently defined terms of “Inter-zonal” and “Intra-zonal” congestion
- Accepts self-scheduling of resources (there are no bids, just MW schedules)
- Includes a Security Constrained Unit Commitment (SCUC) with Start-up Cost, Minimum Load Cost, Incremental energy cost, Capacity Bid for A/S

This presentation is kept at the simple level of explaining the optimization of energy only in the determination of the LMP and does not into any details about the co-optimization or SCUC
LMP Overview

- Locational marginal pricing (LMP) is a mechanism for using market-based prices for managing transmission congestion
  - Prices are determined by the bids/offers submitted by market participants
  - The charge for transmission usage is the incremental cost of the redispatch required to accommodate that transmission usage
  - If there is no transmission congestion, the charge for transmission usage would be zero (except for other charges to recover portions of the embedded cost of the transmission grid, etc.)
LMP Overview

- Locational marginal prices differ by location when transmission congestion occurs
  - Transmission congestion prevents energy from low-cost generation from meeting all loads and clearing the market
  - Some of this low-cost generation must be constrained down/off, lowering the market price of generation in the unconstrained area
  - Higher-cost generation must be constrained on to serve load in the constrained area, raising the market price of generation in the constrained area
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LMP Definition

- Locational Marginal Price (LMP) is the marginal cost of supplying, at least cost, the next increment of electric demand at a specific location (node) on the electric power network, taking into account both supply (generation/import) bids and demand (load/export) offers and the physical aspects of the transmission system including transmission and other operational constraints.

- Important concepts to note
  - Marginal cost
  - Least cost
  - Supply bids and demand offers
  - Physical aspects of the transmission system

- Define the term “Nodal Price” to be an LMP at a specific node.

- The term LMP can also be generically used for the price of a load aggregation point.
LMP Definition

- The definition and calculation of an LMP is based on
  - Economic theory
    - Marginal cost (aka variable cost)
    - Least cost
    - Bids (supply curves) and Offers (demand curves)
  - Power system operational practices
    - Physical aspects of the transmission system
    - Operating the power system based on operating criteria
      - Operating procedures and constraints
LMP Definition

- Marginal cost concept
  - Based on the marginal cost of the “total system cost”
- The *total system cost* is similar to *social surplus* (aka total surplus)
  - Social surplus is a standard economic theory term and is calculated from
    - Supplier surplus (based on supply curves)
    - Consumer surplus (based on demand curves)
- Review of basic economic theory
LMP Definition – Basic Economic Theory

Supply Curve
- Represents the marginal (Variable) cost of supply. \( P(Q^*) \) represent the \textbf{minimum} price-per-unit the supplier is willing to be paid to produce the next increment of \( Q \) at the point of \( Q^* \).
- The area under the curve up to \( Q^* \) represent the total cost ($/h) to produce the quantity \( Q^* \) (not considering a constant cost factor).
- Monotonically increasing: \( P(Q_1) \geq P(Q_2), Q_2 > Q_1 \)

Demand Curve
- Represents the marginal (Variable) benefit of demand. \( P(Q^*) \) represent the \textbf{maximum} price-benefit-per-unit the consumer is willing to pay to consume the next increment of \( Q \) at the point of \( Q^* \).
- The area under the curve up to \( Q^* \) represent the total benefit ($/h) to consume the quantity \( Q^* \) (not considering a constant cost factor).
- Monotonically decreasing: \( P(Q_1) \leq P(Q_2), Q_2 > Q_1 \)
LMP Definition – Basic Economic Theory

**Supplier Surplus**
- If the total quantity of $Q^*$ is priced at $P(Q^*)$, the solid area shown is Supplier Surplus and represents the extra revenue above what is required to produce the quantity $Q^*$.
- Total revenue for $Q^*$ is at least $P(Q^*) \times Q^*$. Price may be $> P(Q^*)$. Assume total revenue is $P(Q^*) \times Q^*$. Total revenue minus total cost to supply (lower triangle) = area of upper triangle (i.e., Supplier Surplus)

**Consumer or Demand Surplus**
- If the total quantity of $Q^*$ is priced at $P(Q^*)$, the solid area shown is Consumer Surplus and represents the extra benefit the consumer saves to consume the quantity $Q^*$.
- Total payment for $Q^*$ is at $P(Q^*) \times Q^*$ or less. Price may be $< P(Q^*)$. Assume total payment for $Q^*$ is $P(Q^*) \times Q^*$ (bottom square). Total benefit (upper triangle + lower square) minus total payment (lower square) = area of upper triangle (i.e., Consumer Surplus)
Total Surplus (aka Societal Surplus) = Supplier Surplus + Consumer Surplus

-To reach a clearing price and create an efficient transaction, the total surplus is maximized under the condition that total supply equal total demand.
LMP Definition – Basic Economic Theory

- To achieve efficiency and match supply quantities with demand quantities and create a “price”, the total surplus is maximized
  - Maximizes the benefits to both suppliers and consumers
- Note that \( P(Q^*) \) represents the price for
  - the next increment of supply to be provided
  - the next increment of demand to be consumed
  - \( P(Q^*) \) is the *marginal* clearing price
- LMPs are calculated using these basic economic theory principles
LMP Definition – Basic Economic Theory

- Note that at a clearing quantity and clearing price, the Total Surplus is equal to total consumer benefit (area under the demand offer) minus the total cost to supply (area under the supply bid).

- The values of $Q^*$ and $P(Q^*)$ can be determined from an optimization formulation:
  - Objective function
    - Maximize: Total Surplus
  - Constraints
    - Total supply quantity = Total demand quantity
LMP Definition – Basic Economic Theory

Objective function can be written as

- $\text{Max} \ (\ \text{Total Surplus})$
- $\text{Max} \ (\ \text{total consumer benefit minus the total cost to supply})$
- $\text{Max} \ (\ \text{area under the demand offer minus the area under the supply bid})$
- $\text{Min} \ (\ \text{area under the supply bid minus area under the demand offer})$
  - The order of subtraction is flipped around so Max changes to Min
  - area under the supply bid minus area under the demand offer is equal to negative of the Total Surplus
- $\text{Min} \ (\ \text{area under the supply bid plus (the negative of the area under the demand offer})$
  - The negative of the area under the demand offer can be modeled assuming the demand MWhs are negative MWhs
  - The reference direction is an injection into the system and is positive, thus demand or withdrawal from the system is negative
**LMP Definition**

A sample supply bid as shown below. The supply cost is the area under the curve up to the Dispatch MW. Area under the curve represents $(\$/\text{MWh}) \times (+\text{MW}) = \$/\text{h}.$

A sample demand bid as shown below. The consumer benefit is the area under the curve up to the Dispatch MW. Area under the curve represents $(\$/\text{MWh}) \times (-\text{MW}) = -\$/\text{h}.$ The MWs are negative meaning withdrawal (not injection) from the system.

\[
\text{Area} = \text{Price}_1 \times \text{MW}_1 + \text{Price}_2 \times (\text{Dispatch MW} - \text{MW}_1)
\]
LMP Definition

- LMP calculation is based on
  - Objective Function
    - Min (negative of the Total Surplus)
    - Min (Total System Cost)
    - Note that Total System Cost is similar to Total Surplus and in this presentation it is defined as the negative of the Total Surplus
  - Subject to:
    - Total supply = total load (this ignores losses)
    - Adhering to all operational constraints
      - e.g., interface flow limits
  - The LMPs are the marginal prices, which are price sensitivities that are produced at the solution of the optimization problem
LMP Definition

- The calculation of the LMPs are much more complicated than the simple one supply curve, one demand curve example because transmission constraints cannot be violated.

- If transmission constraints were not enforced then all supply curves could be aggregated and all demand curves could be aggregated and the simple methodology could be applied to determine the system wide clearing quantity and price.

- Since transmission constraints are enforced, marginal prices are on a per node basis rather than a system wide basis.
LMP Definition

- Simple or Layman’s Definition of LMP on a per nodal basis
  - This is a more intuitive way of understanding the LMP at a node
  - LMP for energy at each node is determined by adding 1 MW of fixed load at that node and determining
    - The least change of total system cost (this provides the overall marginal price for supplying one more unit of quantity, i.e., the 1 MW)
    - While satisfying all transmission and other operational constraints
  - LMP = Change in total system cost divided by 1 MW = change in total system cost
LMP Definition

- Simple two bus example with a generator (supply) bid and a load (demand) offer
  - Assume no losses
  - Assume the line is not enforced so that there will not be congestion

\[
\begin{align*}
\text{$/MWh} & \quad \text{MW} \\
10 & \quad 200 & + & \text{MW} \\
\text{$/MWh} & \quad \text{-MW} & \quad 100 & \quad 50
\end{align*}
\]
LMP Definition

- Load is willing to pay up to $50/MWh for 100 MW of load
- Generation is willing to sell up to 200 MW at no less than $10/MWh

Optimize

- Minimize the objective function
  - Minimize (Total system cost)
- Such that total supply equals to total load
**LMP Definition**

- **Supply cost** = area under generation bid
  - $10/MWh \times \text{Supply MW}

- **Demand offer** = area under load offer
  - $50/MWh \times (-\text{Demand MW})
  - Note the negative sign on the Demand MW

- **Optimization formulation**
  - Min ($10/MWh \times \text{Supply MW} + 50/MWh \times (-\text{Demand MW})$
  - Such that \text{Supply MW} = \text{Demand MW}

- **The solution comes out to be**
  - \text{Supply MW} = \text{Demand MW} = 100 \text{ MW}
  - Total system cost is
    - $10/MWh \times 100 \text{ MW} + 50/MWh \times (-100 \text{ MW})$
    - $1,000 - 5,000 = -$4,000/h
LMP Definition

- What is the LMP at the load node?
- Note that since the line constraint is not enforced and losses are ignored it is effectively the same as the load and generation being located at the same bus.
- The supply curve and the demand curve can be put onto the same graph to determine the marginal clearing price, but let’s go through the Layman’s definition process and try to determine the marginal price that way.
LMP Definition

- Add one additional MW to the load bus
  - This MW is *not* associated with the load bid
  - There is no bid for this additional MW, it is fixed

- Need to ensure that the total generation equals total load – select 2 options to ensure this
  - Option 1: This 1 MW can be served by the generator
    - Total generation = 100 + 1 = 101 MW
    - Total load = 100 + 1 = 101 MW
  - Option 2: This 1 MW can be served by reducing the load bid by 1 MW and leaving the generation at the same dispatch level
    - Total generation = 100 MW
    - Total load = (100 – 1) + 1 = 100 MW
LMP Definition

- Total system costs for each option

  **Option 1:**
  - Supply cost = $10/MWh × 101 MW = $1,010/h
  - Demand cost = $50/MWh × (-100 MW) = -$5,000/h
  - Total cost = 1,010 – 5,000 = -$3,990/h

  **Option 2:**
  - Supply cost = $10/MWh × 100 MW = $1,000/h
  - Demand cost = $50/MWh × (-99 MW) = -$4,950/h
  - Total cost = 1,000 – 4,950 = -$3,950/h
LMP Definition

- What is the minimum of the two changes in total system cost
- Total system cost is minimized my selecting option 1
  - -$3,990/h is less than -$3,950/h
  - Any other combination of increasing generation by x (0 < x < 1) MW and decreasing the load bid by (1-x) MW will lead to a total system cost greater than -$3,990/h and less than -$3,950/h
- Change in cost for option 1
  - -$3,990/h – (-$4,000/h) = $10/h
- LMP for load bus is
  - $10/h divided by 1 MW = $10/MWh
  - Same solution as plotting the two curves on the same graph
Agenda

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- LMP Definitions
- LMP Characteristics
- LMP Settlements Overview
- LMP Observations
- BREAK
- LMP Examples (unconstrained cases)
- LMP Examples (constrained cases)
LMP Characteristics

- LMPs are determined from the result of a security-constrained least-cost dispatch
  - Optimal Power Flow (OPF)
  - This is part of the Integrated Forward Market design
- LMPs may differ between locations due to
  - Transmission congestion
  - Losses
- Under Locational Marginal Pricing, the charge for transmission use in terms of congestion and losses is the difference between the LMPs at the receipt and delivery locations.
LMP Characteristics

- Unconstrained conditions (no congestion)
  - Single Market Clearing Price (assuming no losses)
    - This is unconstrained economic dispatch

- Constrained conditions (congestion exists)
  - The marginal cost of energy varies by location as low cost supply cannot reach all demand
  - LMPs reflect increased cost to deliver energy when inadequate transmission exists
  - LMPs can be quite different from the single unconstrained economic dispatch price due to costs to deliver energy from the marginal generating units to load buses
LMP Characteristics

- Aggregated Load Point Pricing
  - The LMP for an aggregated load point will be a load weighted average of the nodal LMPs of the aggregated load point.
  - The load weights will be based on Load Distribution Factors.
  - LMPs for aggregated load points are usually used to settle loads while LMPs at the nodal level are usually used to settle generation.
LMP Characteristics

Nodal Price Components

Each nodal price can be decomposed into 3 components

- Marginal cost at a reference bus
- Marginal cost of transmission losses (thermal losses in the transmission lines)
- Marginal cost of transmission system congestion due to binding constraints, e.g., binding transmission line constraints
LMP Characteristics

- **Nodal Price Components**
  - These components are determined through a software algorithm.
  - In general, the 3 components are calculated as separate variables in the algorithm and are then combined to get the nodal price.
LMP Characteristics

The LMP (nodal price) at Bus \( i \) can be calculated using the following equation

\[
\lambda_i = \lambda_{\text{Ref}} - L_i \times \lambda_{\text{Ref}} - \sum_j (\mu_j \times \text{SF}_{ji})
\]

where

- \( \lambda_i \) = Nodal price at bus \( i \)
- \( \lambda_{\text{Ref}} \) = Nodal price at the reference bus
- \( L_i \) = Marginal loss factor at bus \( i = (\partial \text{Ploss}/\partial P_i) \), \( P_i \) is injection at bus \( i \) and \( \text{Ploss} \) is the system losses
- \( \mu_j \) = Shadow price of constraint \( j \)
- \( \text{SF}_{ji} \) = Shift factor for real load at Bus \( i \) (the reference bus (ref) is the reference bus for this shift factor) on constraint \( j \)
LMP Characteristics

Nodal Price Components

\[ \lambda_i = \lambda_{Ref} - L_i \times \lambda_{Ref} - \sum_j (\mu_j \times SF_{ji}) \]

- Marginal Cost at the reference bus
- Marginal Cost of losses from the reference bus to bus \( i \)
- Marginal Cost of transmission congestion from the reference bus to bus \( i \)

\[ \lambda_i = \lambda_{Ref} + \lambda_{Lossi} + \lambda_{Congestioni} \]
LMP Characteristics

\[ \lambda_{\text{Ref}}: \text{The marginal cost at a reference bus} \]

- The nodal price at the reference bus
- The nodal price at each bus \( i \) shares this same component
- This nodal price includes an implicit congestion component
  - That is, the nodal price at the reference bus is the least marginal cost of supplying the next increment of load at the reference bus taking into account the physical aspects (i.e., constraints) of the transmission network (i.e., potential congestion)
  - No implicit loss component, the loss components are calculated with respect to the reference bus
- This is generally not the nodal price if the system is unconstrained
LMP Characteristics

\[ \lambda_{\text{Lossi}} = - L_i \times \lambda_{\text{Ref}}: \text{The marginal cost of losses from the reference bus to bus } i \]

\[ + L_i \times \lambda_{\text{Ref}}: \text{The marginal cost of losses from bus } i \text{ to the reference bus} \]

\[ L_i = (\frac{\partial P\text{loss}}{\partial P_i}) \]

- For example, withdraw 1 MW at the reference bus and balance this with an injection at bus \( i \), \( \Delta P_i \)

- there will be some change in the losses, \( \Delta \text{Loss} \)

- To keep a power balance, the injection at bus \( i \) will be
  \[ \Delta P_i = \Delta \text{Loss} + 1 \text{ MW} \]

\[ (\frac{\partial P\text{loss}}{\partial P_i}) = (\frac{\Delta \text{Loss}}{\Delta P_i}) \]

- \( L_i \) may be positive if there is an increase in loss or negative if there is a decrease in losses (for counter flow that decreased flow on transmission lines)
LMP Characteristics

\[ \lambda_{\text{Congestion}i} = - \sum_j (\mu_j \times SF_{ji}) \]: marginal cost of transmission congestion from the reference bus to bus \( i \)

\[ + \sum_j (\mu_j \times SF_{ji}) \]: marginal cost of transmission congestion from bus \( i \) to the reference bus

\( \mu_j \) is the shadow price ($/MWh)

- Associated with a binding constraint
- A binding constraint, for example, is when the flow on the interface is at the limit of the interface
- The value is equal to the incremental change in the system cost divided by an incremental change in the constraint limit

\( SF_{ji} \) is incremental amount of power flow on constraint \( j \) when an additional unit of power is injected at bus \( i \) and withdrawn at the reference bus
LMP Characteristics

Sample Prices for LMP components

\[ \lambda_i = \lambda_{Ref} - L_i \times \lambda_{Ref} - \sum_j (\mu_j \times SF_{ji}) \]

\[ \lambda_i = \lambda_{Ref} + \lambda_{Lossi} + \lambda_{Congestion} \]

$40$/MWh = $50$/MWh - $1$/MWh - $9$/MWh
LMP Characteristics

- $40/\text{MWh} = \text{Marginal Cost at bus } i$
- $50/\text{MWh} = \text{Marginal Cost at the reference bus}$
- $-1/\text{MWh} = \text{Marginal Cost of losses from the reference bus to bus } i$
  -+$1/\text{MWh} = \text{Marginal Cost of losses from bus } i \text{ to the reference bus}$
  -$1/\text{MWh} = - L_i \times \lambda_{\text{ref}} = - L_i \times 50/\text{MWh}$
  - Implies $L_i = - \left(\frac{-1/\text{MWh}}{50/\text{MWh}}\right) = +0.02$
  - $L_i = 0.02$ means an increase in losses for transferring power from bus $i$ to the reference bus
  - For every 1 MWh that is transferred from bus $i$ to the reference bus will effectively incur $1$ of charge for the cost to supply the needed MWhs to make up for the losses
- $-9/\text{MWh} = \text{Marginal Cost of transmission congestion from the reference bus to bus } i$
  -+$9/\text{MWh} = \text{Marginal Cost of transmission congestion from bus } i \text{ to the reference bus}$
LMP Components (NYISO LBMPs)

- Sample LMP data from NYISO for a given hour on a certain day
  - Shows total LMP and components
  - NYISO calls it Locational Bus Marginal Prices (LBMP)

- Observations
  - Reference bus LMP is the same for all buses
  - The loss component is relatively small and is sometimes negative
    - Negative loss component implies a reduction in losses from transferring power from bus $i$ to the reference bus
LMP Components (NYISO LBMPs)

NYISO (June 5, 2002, Hour 14)

**LBMP components**
- Energy
- Congestion
- Losses
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LMP Settlements

- Settlements
  - Generators and imports get paid based on the locational LMP, i.e., paid based upon the LMP at the location of the generator or import
  - Exports pay based on the LMP at export location bus
  - Loads will generally be settled at load aggregation point price
  - Transactions pay congestion charges based on the differential in Sink (load/export) LMP congestion component and Source (generator/import) LMP congestion component
LMP Settlements

Settlements

- The charge for selling 1 MW at A and buying 1 MW at B is (neglecting losses) exactly the same as the charge for transmission from A to B

  \[ \lambda_A = \lambda_{\text{Ref}} + \lambda_{\text{LossA}} + \lambda_{\text{CongestionA}} \]

  \[ \lambda_B = \lambda_{\text{Ref}} + \lambda_{\text{LossB}} + \lambda_{\text{CongestionB}} \]

  \[ \lambda_{\text{LossA}} = \lambda_{\text{LossB}} = 0 \]

- The rate for transmission from A to B is the congestion congestion rate and is priced at

  \[ \lambda_B - \lambda_A = \lambda_{\text{CongestionB}} - \lambda_{\text{CongestionA}} \]

- Transmission service for bilateral transactions will therefore be priced on the same basis as, and consistent with, energy market transactions
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LMP Observations

- The locational price at a bus is not necessarily equal to the bid of any single generator
  
  - The LMP price is the cost of meeting the next increment of load at a particular location through least-cost dispatch of the entire system, not merely generation at that location

- In most situations, but not all, the LMP price at most locations will be determined by the bids of generators at other locations on the transmission system
LMP Observations

- A generator’s bid will generally set the locational price at its location when the generator capacity segment is only partially dispatched (unless it is at its minimum, or being held down to provide regulation, spinning reserves, or voltage support).
  - This is a marginal unit
- If a generator capacity segment is fully dispatched, the locational price that it is paid will be determined by the bids of other generators and will be greater than or equal to the generator’s energy bid for that capacity segment.
LMP Observations

- If a generator capacity segment is not dispatched, the locational price will be less than its energy bid.

- The congestion component of LMPs can differ between two buses even if the flow on the line that directly connects these two buses is not binding (i.e., flow < limit).

- The congestion component of LMPs can vary over the entire system if there is just one binding constraint.
  - This can happen in a looped system where parallel paths are present.
  - Unlike the current zonal based congestion management configuration of a zonal model with radial connections.
LMP Observations

- The LMP at a bus may be larger than the largest bid over all the resources in the system.
- The LMP at a bus may be negative.
  - The increase of load on that bus may decrease flows and decrease the total system cost.
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LMP Examples

There are 3 types of examples that will be presented:

All three types of examples ignore losses (an example with losses will be covered in the Settlements Presentation)

1. Market Clearing Price with Economic dispatch
   - Do not take into account operational constraints
   - Single Market Clearing Price

2. Nodal Price calculation on a system where constraints are not binding
   - All Nodal prices are equal (assuming no transmission loss)
LMP Examples

- (3) Nodal Price calculation on a system where constraints are binding
  - Under constrained conditions, the marginal cost of energy varies by location as low cost supply cannot reach all demand
  - LMPs reflect increased cost to deliver energy when inadequate transmission exists
  - Under constrained conditions, LMPs can be quite different from the economic dispatch rates due to costs to deliver energy from the marginal generating units to load buses
Example 1A - Economic Dispatch

Least expensive generators are dispatched. Merit order of generators from least expensive to most expensive. No constraints are enforced – No congestion.

- MW $10
  Capacity 300 MWs
  300MWs @ $10

- MW $15
  Capacity 200 MWs
  199MWs @ $15

- MW $20
  Capacity 200 MWs
  Not Dispatched

Load
499 MW

Market Operator

LMP for all settlement nodes is $15
Example 1B - Economic Dispatch

Highest Cost Generator Sets Price

- MW $10
  Capacity 300 MWs

- MW $15
  Capacity 200 MWs

- MW $20
  Capacity 200 MWs

I need MWs. Sale goes to the lowest bidder with capacity. Going once....

Load

599 MW

Market Operator

$20

LMP for all settlement nodes is $20

99 MWs@ $20

200 MWs@ $15

300 MWs@ $10
Examples 2 and 3 – Flow of Examples

- Nodal Price calculation using the 3 bus system
- The flow of these examples are
  - Set load at a level where there is no congestion
    - Calculate nodal prices
  - Increase load to an appropriate point where transmission congestion occurs and generation re-dispatch is required
    - Calculate nodal prices
Examples 2 and 3 – System Set Up

The LMP examples will use the following 3 bus system:

- 2 Load buses: Bus B and Bus C
- 2 Generators: Bus A and Bus C
- All three transmission lines have equal impedances and no losses

Energy bid is $10/MWh

Capacity = 300 MW

Energy bid is $20/MWh

0 MW @ $20 Bid

Capacity = 400 MW

Capacity = 200 MW
First understand the flow of power in the system
All three transmission lines have equal impedances and no losses

1 MW injection at bus A and Withdrawal of 1 MW at bus C

1 MW injection at bus B and Withdrawal of 1 MW at bus C
Examples 2 and 3 – System Power Flows

1 MW injection at bus A and Withdrawal of 1 MW at bus B
Example 2A – Unconstrained with low load level

200 MW of load at Bus C, 0.0 MW load at Bus B: Total load = 200 MW

- Final LMP Prices
  - Bus A = $10
  - Bus B = $10
  - Bus C = $10

Bid is $10/MWh
Final Dispatch = 200 MW

\[
\text{Flow}_{AB} = \text{Flow}_{BC} = \frac{1}{3} \times 200 \text{ MW} = 67\text{MW}
\]
\[
\text{Flow}_{AC} = \frac{2}{3} \times 200 \text{ MW} = 133\text{ MW}
\]
Example 2A – Unconstrained with low load level

- Determine the LMP at each bus
  - Add 1 MW of load at each bus and determine the change in the total system cost taking into consideration transmission constraints

- Add 1 MW of Load at:
  - Bus A: can be served by G1 for an extra cost of $10 for that MWh, LMP = $10 MWh
  - Bus B: can be served by G1 for an extra cost of $10 for that MWh, LMP = $10 MWh
  - Bus C: can be served by G1 for an extra cost of $10 for that MWh, LMP = $10 MWh
Example 2B – Unconstrained with Medium Load Level

200 MW of load at Bus C and increase load at Bus B to 50 MW: Total load = 250 MW

- Final LMP Prices
  - Bus A = $10
  - Bus B = $10
  - Bus C = $10

Final LMP Prices

Bid is $10/MWh
Final Dispatch = 250 MW

Flow_{AB} = (1/3 \times 200 MW) + (2/3 \times 50) = 100 MW
Flow_{AC} = (2/3 \times 200 MW) + (1/3 \times 50) = 150 MW
Flow_{BC} = (1/3 \times 200) - (1/3 \times 50) = 50 MW (or 100 - 50 Load = 50 MW)
Example 2B – Unconstrained with Medium Load Level

Determine the power flow in the system

Split up the Generation to match each of the loads

50 MW and 200 MW injection at bus A
Withdrawal of 50 MW at bus B
Withdrawal of 200 MW at bus C
Example 2B – Unconstrained with Medium Load Level

- Determine the LMP at each bus
  - Add 1 MW of load at each bus and determine the change in the total system cost taking into consideration transmission constraints

- Add 1 MW of Load at:
  - Bus A: can be served by G1 for an extra cost of $10 for that MWh, LMP = $10 MWh
  - Bus B: can be served by G1 for an extra cost of $10 for that MWh, LMP = $10 MWh
  - Bus C: can be served by G1 for an extra cost of $10 for that MWh, LMP = $10 MWh
## Example 2B - Settlement Details

### LOAD PAYMENTS

<table>
<thead>
<tr>
<th>Location</th>
<th>LMP</th>
<th>Load (MWh)</th>
<th>Total Charge</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bus B</td>
<td>$10.00</td>
<td>50</td>
<td>$500.00</td>
</tr>
<tr>
<td>Bus C</td>
<td>$10.00</td>
<td>200</td>
<td>$2,000.00</td>
</tr>
</tbody>
</table>

**Total**  $2,500.00

### GENERATION CREDITS

<table>
<thead>
<tr>
<th>Location</th>
<th>LMP</th>
<th>Generation (MWh)</th>
<th>Total Credit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bus A</td>
<td>$10.00</td>
<td>250</td>
<td>$2,500.00</td>
</tr>
</tbody>
</table>

**Total**  $2,500.00

### Collections

- **Total**  $2,500.00

### Payments

- **Total**  $2,500.00

### Net

- **Total**  $0.00
Example 2C - Unconstrained with High Load Level and Transmission Violation

325 MW of load at Bus C and 75 MW of load at Bus B: Total load = 400 MW
Assume line constraints are not enforced

- Final LMP Prices
  - Bus A = $10
  - Bus B = $10
  - Bus C = $10

Bid is $10/MWh
Final Dispatch = 400 MW

Flow $\text{AB} = \left(\frac{1}{3} \times 325 \text{ MW}\right) + \left(\frac{2}{3} \times 75 \text{ MW}\right) = 158 \text{ MW}$

Flow $\text{AC} = \left(\frac{2}{3} \times 325 \text{ MW}\right) + \left(\frac{1}{3} \times 75 \text{ MW}\right) = 242 \text{ MW}$

Flow $\text{BC} = \left(\frac{1}{3} \times 325 \text{ MW}\right) - \left(\frac{1}{3} \times 75 \text{ MW}\right) = 83 \text{ MW (or 158 - 75 Load = 83 MW)}$

LMP = $10$

Constraint Violated !!!

Bid is $20/MWh
Final Dispatch = 0 MW

Capacity = 400 MW

Capacity = 300 MW

75 MW Load

83 MW

325 MW Load
Example 2C - Unconstrained with High Load Level and Transmission Violation

- Determine the LMP at each bus
  - Add 1 MW of load at each bus and determine the change in the total system cost taking into consideration transmission constraints
- Add 1 MW of Load at:
  - Bus A: can be served by G1 for an extra cost of $10 for that MWh, LMP = $10 MWh
  - Bus B: can be served by G1 for an extra cost of $10 for that MWh, LMP = $10 MWh
    - The limit on line segment A-C is not being enforced
  - Bus C: can be served by G1 for an extra cost of $10 for that MWh, LMP = $10 MWh
    - The limit on line segment A-C is not being enforced
### Example 2C - Settlement Details

#### LOAD PAYMENTS

<table>
<thead>
<tr>
<th>Location</th>
<th>LMP</th>
<th>Load (MWh)</th>
<th>Total Charge</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bus B</td>
<td>$10.00</td>
<td>75</td>
<td>$750.00</td>
</tr>
<tr>
<td>Bus C</td>
<td>$10.00</td>
<td>325</td>
<td>$3,250.00</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td></td>
<td><strong>$4,000.00</strong></td>
</tr>
</tbody>
</table>

#### GENERATION CREDITS

<table>
<thead>
<tr>
<th>Location</th>
<th>LMP</th>
<th>Generation (MWh)</th>
<th>Total Credit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bus A</td>
<td>$10.00</td>
<td>400</td>
<td>$4,000.00</td>
</tr>
</tbody>
</table>

**Collections** $4,000.00  
**Payments** $4,000.00  
**Net** $0.00
Agenda

- LMP Overview
- LMP Definitions
- LMP Characteristics
- LMP Observations
- BREAK
- LMP Settlements
- LMP Examples (unconstrained cases)
- LMP Examples (constrained cases)
Example 3 - Redispatch to Alleviate Constraint Violation

- In the last example there was a line overload on line segment A-C
- Flow on line segment A-C is 242 MW
  - Based on dispatch of G1 (Gen at bus A) of 400 MW
- The line limit for line segment A-C is 200 MW
- Need to re-dispatch resources to alleviate the constraint violations
- Available resources with bids are
  - G1 (generation at bus A) (bid is $10/MWh)
  - G2 (generation at bus C) (bid is $20/MWh)
- Need to curtail cheap energy and dispatch more expensive energy to alleviate the constraint
- In this case, curtail G1 and dispatch G2
Example 3 - Redispatch to Alleviate Constraint Violation

- Dispatch solution to alleviate the constraint violation
- Curtail G1 and dispatch G2 to a level that reduces the flow on line segment A-C to a level of 200 MW
- The required dispatch of G2 is out of economic merit order
- How to determine this curtailment/dispatch change?
Example 3 - Redispatch to Alleviate Constraint Violation

- The net load at Bus C would be
  - $325 - G_2$ output

- The generation at bus A would serve the remaining load
  - $G_1$ output = $(325 - G_2$ output) + 75 MW of load at bus B

- Split the $G_1$ into two parts to determine the flows on the lines
  - 75 MW to serve the load at bus B
  - $325 - G_2$ to serve the net load at bus C
Example 3 - Redispatch to Alleviate Constraint Violation

Determine the power flow in the system

Split up the Generation to match each of the loads

\[
\begin{align*}
&\text{325 - G2 MW} \\
&\text{75 MW}
\end{align*}
\]

\[
\begin{align*}
&\text{(2/3)} \times 75 \\
&(1/3) \times (325 - \text{G2}) \\
&\text{(2/3)} \times (325 - \text{G2}) \\
&(1/3) \times 75 \\
&(1/3) \times 75
\end{align*}
\]

\[
\begin{align*}
&\text{75 MW} \\
&\text{(325 - G2) MW}
\end{align*}
\]
Example 3 - Redispatch to Alleviate Constraint Violation

- Set the flow on line segment A-C to be 200 MW
  - \((2/3) \times (325 - G2) + (1/3) \times 75 = 200\) MW
    - \((2/3) \times (325 - G2) = 200 - (1/3) \times 75\)
    - \(325 - G2 = (3/2) \times (200 - (1/3) \times 75)\)
    - \(G2 = 325 - (3/2) \times (200 - (1/3) \times 75)\) MW
    - \(G2 = 325 - (3/2) \times (200 - 25)\) MW
    - \(G2 = 325 - (3/2) \times (175)\) MW
    - \(G2 = 325 - 262.5\) MW
  - \(G2 = 62.5\) MW
  - \(G1 = (325 - G2) + 75\) MW
    - \(G1 = (325 - 62.5) + 75\) MW
  - \(G1 = 337.5\) MW
Example 3 - After constraint is relieved

325 MW of load at Bus C and 75 MW of load at Bus B: Total load = 400 MW
Line constraints are enforced

Final LMP Prices
Bus A = $10
Bus B = ??
Bus C = $20

325 MW of load at Bus C and 75 MW of load at Bus B: Total load = 400 MW
Line constraints are enforced

Flow \( AB \) = 338 MW - 200 MW = 138 MW
Flow \( AC \) = 200 MW (Constraint)
Flow \( BC \) = \((1/3 \times 263) - (1/3 \times 75)\) = 63 MW (or 138 - 75 Load = 63 MW)
Example 3 - Determine the LMPs

- Determine the LMP at each bus
  - Add 1 MW of load at each bus and determine the change in the total system cost taking into consideration transmission constraints

- Add 1 MW of Load at:
  - Bus A: can be served by G1 for an extra cost of $10 for that MWh, LMP = $10 MWh
  - Bus C
    - Cannot be served by G1 since any dispatch by G1 to cover an additional 1 MW of load at bus C would increase flow on line segment A-C and violate the 200 MW constraint
    - Needs to be served by G2 for an extra cost of $20 for that MWh
      - LMP = $20 MWh
  - Bus B ?
Example 3 - Determining the Bus B LMP

LMP at bus B

- Add 1 MW of load at bus B

Options to serve this 1 MW of load

- Serve this 1 MW of load from G1
  - This would violate the constraint on line segment A-C
  - 1/3 of this MW would go over line segment A-C

- Serve this 1 MW of load from G2
  - This would reduce the flow on line segment A-C by 1/3 MW and would not violate any other constraints
  - The resulting LMP at bus B would be $20/MWh, i.e., the additional cost to serve this 1 MW at bus B
  - However, is this dispatch the least cost dispatch?
  - Answer is NO

- Serve this 1 MW of load from a combination of G1 and G2
  - Creates no violations
  - Serves this additional 1 MW at least cost since G1 is cheaper than G2 a combination of G1 and G2 is cheaper than just using G2
Example 3 - Determining the Bus B LMP

- Serve the additional 1 MW of load at bus B by a combination of G1 and G2 output
- Keep the flow on line segment A-C at 200 MW
  - Or make sure the net contribution of G1 and G2 on line segment A-C is 0 MW
- Determine the additional output from G1 and G2
- How to do this?
- Look at the flows on the system with a combination of additional G1 and G2 output with
  - $\Delta G1 + \Delta G2 = 1$ MW (the 1 MW of load at Bus B)
Example 3 – Determining the Bus B LMP

Determine the incremental power flow in the system with an additional 1 MW at bus B

\[
\Delta G_1 \times (1/3) \times (1 - \Delta G_1) \\
(2/3) \times \Delta G_1 \\
(1/3) \times \Delta G_1 \\
(1/3) \times (1 - \Delta G_1) \\
(1/3) \times (1 - \Delta G_1) \\
\Delta G_2 = 1 - \Delta G_1
\]

\[
\Delta G_1 + \Delta G_2 = 1 \text{ MW} \\
\Delta G_2 = 1 - \Delta G_1
\]
Example 3 – Determining the Bus B LMP

- Keep the flow on line segment A-C at 200 MW
  - Or make sure the net contribution of G1 and G2 on line segment A-C is 0 MW

- The additional net flow on line segment A-C in the A to C direction is
  - \( \frac{1}{3} \times \Delta G_1 - \left( \frac{1}{3} \times (1 - \Delta G_1) \right) \)

- Set this additional flow to zero and solve for \( \Delta G_1 \)
  - \( \frac{1}{3} \times \Delta G_1 - \left( \frac{1}{3} \times (1 - \Delta G_1) \right) = 0 \)
  - \( \frac{2}{3} \times \Delta G_1 = \frac{1}{3} \)
  - \( \Delta G_1 = \frac{1}{2} \) MW

- Since \( \Delta G_1 + \Delta G_2 = 1 \) MW
  - \( \Delta G_2 = \frac{1}{2} \) MW
Example 3 – Determining the Bus B LMP

- The change in the system cost would be
  - \((1/2 \text{ MW}) \times \$10/\text{MWh} + (1/2 \text{ MW}) \times \$20/\text{MWh} = \$15/\text{h}\)
- LMP at bus B = \((\$15/\text{h}) / (1 \text{ MW})\)
- LMP at bus B = \$15/\text{MWh}
- Note here that the LMP is neither of the two bid values of 10/\text{MWh} or \$20/\text{MWh} but a combination of them
Example 3 – Determining the Bus B LMP

- Confirm that the answer meets the criteria for the LMP at bus B
  - The LMP at bus B is $15/MWh and this is determined by $\Delta G_1 = (1/2) \text{ MW}$ and $\Delta G_2 = (1/2) \text{ MW}$ to serve the additional 1 MW of load at bus B
- Remember the definition of LMP at a bus
  - Least marginal system cost to supply next load increment (in this case 1 MW)
  - While satisfying system constraints
- Change the dispatch slightly and check the criteria
  - Perform two sensitivity tests
Example 3 – Determining the Bus B LMP

- Set the following dispatch and check the criteria for the calculation of the LMP at bus B
  - $\Delta G_1 = (0.4) \text{ MW}$
  - $\Delta G_2 = (0.6) \text{ MW}$

- Are the system constraints satisfied?
- The additional net flow on line segment A-C in the A to C direction is
  - $(1/3) \times \Delta G_1 - ((1/3) \times (1 - \Delta G_1))$
  - $(1/3) \times \Delta G_1 - (1/3) \times \Delta G_2$
  - $(1/3) \times (\Delta G_1 - \Delta G_2)$
  - $(1/3) \times (0.4 - 0.6)$
  - $-0.0667 \text{ MW}$

- The flow on line segment A-C in the A to C direction is reduced by $-0.0667 \text{ MW}$
- No congestion and the system constraints are satisfied
Example 3 – Determining the Bus B LMP

- With the dispatch of
  - \( \Delta G_1 = (0.4) \text{ MW} \)
  - \( \Delta G_2 = (0.6) \text{ MW} \)

- What is the marginal cost (i.e., LMP)
  - Change in system cost = \((0.4 \text{ MW}) \times (\$10/\text{MWh}) + (0.6 \text{ MW}) \times (\$20/\text{MWh}) = (\$16/\text{h})\)
  - Marginal cost = \( (\$16/\text{h}) / (1 \text{ MW}) = \$16/\text{MWh} \)

- Change the dispatch and check the criteria again
  - \( \Delta G_1 = (0.6) \text{ MW} \)
  - \( \Delta G_2 = (0.4) \text{ MW} \)

- The additional net flow on line segment A-C in the A to C direction is
  - \((1/3) \times (0.6 - 0.4)\)
  - +0.0667 MW

- This increases the flow on the line and violates the line limit
Example 3 – Determining the Bus B LMP

- Comparing the two sensitivity results
- With the dispatch of
  - $\Delta G_1 = (0.4)$ MW
  - $\Delta G_2 = (0.6)$ MW
  - Feasible with change in system cost of $16/h
  - LMP = $16/MWh
- With the dispatch of
  - $\Delta G_1 = (0.6)$ MW
  - $\Delta G_2 = (0.4)$ MW
  - Infeasible
- With the original dispatch of
  - $\Delta G_1 = (0.5)$ MW
  - $\Delta G_2 = (0.5)$ MW
  - Feasible with change in system cost of $15/h
  - LMP = $15/MWh
  - Best (optimal and feasible) solution, ($15/h) < ($16/h)
Example 3 – LMP at Bus B Determined

Final LMP Prices
Bus A = $10
Bus B = $15
Bus C = $20

Bid is $10/MWh
Final Dispatch = 337.5 MW

LMP = $10

Capacity = 300 MW
138 MW

Flow_{AB} = 338 MW - 200 MW = 138 MW

Flow_{AC} = 200 MW (Constraint)

Flow_{BC} = (1/3 \times 263) - (1/3 \times 75) = 63 MW (or 138 - 75 Load = 63 MW)
## Example 3 - Settlement Details

<table>
<thead>
<tr>
<th>Location</th>
<th>LMP</th>
<th>Load (MWh)</th>
<th>Total Charge</th>
<th>Location</th>
<th>LMP</th>
<th>Generation (MWh)</th>
<th>Total Credit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bus B</td>
<td>$15.00</td>
<td>75</td>
<td>$1,125.00</td>
<td>Bus A</td>
<td>$10.00</td>
<td>338</td>
<td>$3,380.00</td>
</tr>
<tr>
<td>Bus C</td>
<td>$20.00</td>
<td>325</td>
<td>$6,500.00</td>
<td>Bus C</td>
<td>$20.00</td>
<td>62</td>
<td>$1,240.00</td>
</tr>
<tr>
<td>Total</td>
<td>$20.00</td>
<td>325</td>
<td>$7,625.00</td>
<td></td>
<td></td>
<td></td>
<td>$4,620.00</td>
</tr>
</tbody>
</table>

| Collections | $7,625.00 |
| Payments    | $4,620.00  |

| Net Over Collection | -$3,005.00 |
Example 3 – LMP Calculation Using Shift Factors and Shadow Prices

- The LMPs in example 3 were determined using the Layman’s definition.
- There is also the more formal relationship between LMPs, shadow prices and shift factors:

\[ \lambda_i = \lambda_{\text{Ref}} - L_i \times \lambda_{\text{Ref}} - \sum_j (\mu_j \times SF_{ji}) \]
Example 3 – LMP Calculation Using Shift Factors and Shadow Prices

There is one binding constraint which is on line segment A-C in the A to C direction.

- The Shadow Price for this constraint can be determined by increasing the OTC to 201 MW and determine the change in the total system cost.

- Need to go through the mathematics and determine the new values for controls G1 and G2 and then determine the change in the total system cost.

- The equation to solve is
  
  \[
  \frac{2}{3} \times (325 - G2) + \frac{1}{3} \times 75 = 201 \text{ MW}
  \]

  This is the same equation solved earlier where the value of 200 MW was used instead of 201 (= 200 + 1 increment).
Example 3 – LMP Calculation Using Shift Factors and Shadow Prices

- The solution for G2 is 61 MW
  - The OTC increase to 201 MW results in a reduction of this higher priced unit, whose original value was 62.5 MW (a change of -1.5 MW)
- The solution for G1 is 339.0 MW
  - The OTC increase to 201 MW results in an increase in this lower priced unit, whose original value was 337.5 MW (a change of 1.5 MW)
- The change in the total system cost is
  - \(-1.5 \text{ MW} \times $20/\text{MWh} + 1.5 \text{ MW} \times $10/\text{MWh} = -$15/\text{h}\)
- The shadow price is the change in total system cost to the change in constraint value
  - \((-\$15/\text{h})/(200 \text{ MW} - 201 \text{ MW}) = +$15/\text{MWh}\)
- The shadow prices shows that by changing the OTC by 1 MW results in a decrease in total system cost of +$15/\text{MWh}
- Apply the relationship between the LMPs, Shadow Prices and Shift Factors to determine the LMP at Bus B and assume Bus A is the reference Bus
Example 3 – LMP Calculation Using Shift Factors and Shadow Prices

- There are actually 6 constraints with enforced limits (j = 6)
  - #1: Line segment A-B in the A to B direction with limit 300 MW
  - #2: Line segment A-B in the B to A direction with limit 300 MW
  - #3: Line segment B-C in the B to C direction with limit 400 MW
  - #4: Line segment B-C in the C to B direction with limit 400 MW
  - #5: Line segment A-C in the A to C direction with limit 200 MW
  - #6: Line segment A-C in the C to A direction with limit 200 MW

- Relationship between the LMPs, Shadow Prices and Shift Factors

  \[ \lambda_B = \lambda_A - \sum_j (\mu_j \times SF_{ji}) \]

- In this example there is only one non-zero shadow price (for constraint #5) so the above equation reduces to

  \[ \lambda_B = \lambda_A - \mu_{#5} \times SF_{#5,B} \]
Example 3 – LMP Calculation Using Shift Factors and Shadow Prices

- The shift factor SF\(_{#5,B}\) represents the ratio of an injection at Bus B and a withdrawal at the reference Bus (Bus A) to the resulting flow in the direction of the binding constraint (#5).

- An injection at Bus B of 1 MW with a withdrawal at Bus A of 1 MW will cause a flow of \((1/3)\) MW on line-segment A-C but in the C to A direction.

- The flow in the A to C direction, consistent with constraint #5, is \((-1/3)\) MW.
  - \(SF_{#5,B} = (-1/3)\)

- \(\lambda_{B} = \lambda_{A} - \mu_{#5} \times SF_{#5,B}\)

- \(\lambda_{B} = \$10/MWh - (\$15/MWh) \times (-1/3) = \$10/MWh + \$5/MWh = \$15/MWh\)

- This answer is the same as the one determined through the “Layman’s” process.
Any Questions?