California ISO
Shaping a Renewed Future

Demand Response Net Benefits Test

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1. INTRODUCTION

This paper covers the ISO's proposal to fulfill FERC order 745 regarding demand response compensation in the organized wholesale energy market. FERC order 745 requires:

- Demand response (DR) resources will be compensated at full LMP if the LMP is above a threshold price as will be determined by the Net Benefits Test.
- The Net Benefits Test will be performed monthly (by the 15th day) to establish the static monthly threshold price to be used in the next trade month.
- The threshold price is determined by the point where the net benefits of dispatching DR exceeds the marginal cost of DR.
- The net benefit of dispatching DR is estimated based on a representative aggregated supply curve for the trade month.

Per FERC order 745, the representative aggregated supply curve is created in the following way:

- Pick a representative curve of the trade month using previous year's curve.
- Adjust for resource availability.
- Adjust for fuel prices.
- Smooth the curve using numerical methods.

The theory behind the Net Benefits Test is illustrated in Figure 1. In Figure 1, an aggregated supply curve is drawn on the p-q plane, with p representing price and q representing supply quantity. As a convention, consider the aggregated supply curve as price function of supply quantity. A load curve is also drawn on the same p-q plane, which intersects the supply curve at the market clearing equilibrium. Demand response adds elasticity to load. Dispatching demand response will reduce the market clearing price.

- Dispatching an incremental amount (dq) of demand response will reduce the system marginal price (dp) according to the supply curve.
- The benefit to non-DR load for dispatching demand response is q*dp.
- The cost of dispatching demand response is p*dq.
- The net benefit is non-negative if q*dp >= p*dq, or dp/dq >= p/q.
- If there exists a point on the supply curve (p0, q0) with q0 > 0, p0 > 0 and q*dp = p*dq, or equivalently [dp/dq(@q0)] /[p0/q0] = 1 (where @q0 means being evaluated at q0), such that the net benefit is non-negative for all p > p0, then p0 is called the threshold price.
- Demand response should be dispatched only when the clearing price is above the threshold price.

The threshold point condition, q*dp = p*dq, or equivalently (dp/dq) / (p/q) = 1, is a first order necessary condition. It cannot distinguish positive net benefits and negative net benefits for p greater than the threshold price. In the appendix, two theorems are proved to provide second order necessary condition and second order locally sufficient condition for the threshold point. The meaning of theorem 1 (second order necessary condition) is that in order for a point (q0, p0) that satisfies the first order necessary condition to have net non-negative benefits for p>p0, the supply curve must be convex at q0. The meaning of Theorem 2 (second order locally sufficient condition) is that if the supply curve has elasticity equal to one and is strictly convex at a point, then incremental price from this point will result in positive net benefits.

The two theorems further characterize the true threshold point locally beyond the first order necessary condition of elasticity equal to one. When there exists multiple candidate points satisfying the first order necessary condition (elasticity equal to one), the theorems will help find the correct threshold point.

The main body of the ISO's proposal will cover three major aspects:

- How to construct the representative supply curve?
- How to smooth the representative curve?
- How to find the threshold point on the representative curve?

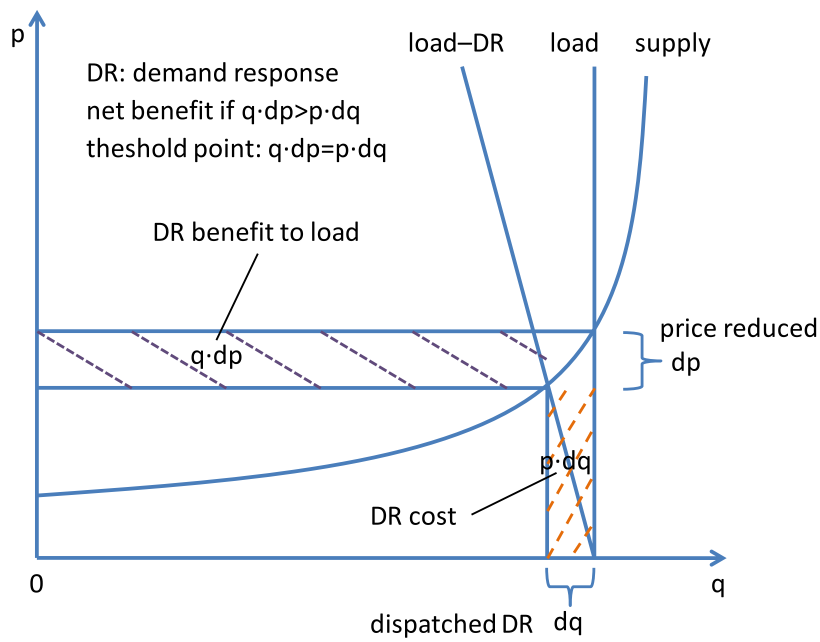

FIGURE 1: DEMAND RESPONSE COST AND BENEFIT

2. CAISO NET BENEFITS TEST DETAILS

2.1 CONSTRUCTING THE REPRESENTITIVE SUPPLY CURVE

The first and most important step of the Net Benefits Test is to construct a representative aggregated supply curve for the trade month, say July 2011. The ISO would publish the Net Benefits Test results by Jun 15th 2011 for July 2011. The construction of the representative supply curves will be based on historical market offers from July 2010, which will be referred to as the reference month. The reference month aggregated supply curve will be called the reference supply curves.

The ISO will construct two reference curves, one for on-peak hours and the other for off-peak hours according to North American Electric Reliability Corporation's (NERC) definition of on-peak and off-peak.[1] The reference supply curves will be constructed based on real-time predispatch (RTPD) mitigated bids from all generation resources including tie-generators, both committed and uncommitted. Import and export bids are excluded.

The reference supply curve must also be adjusted for resource availability. The resource availability can be captured by averaging the hourly reference supply curves over the entire reference month (for every price level, the supply quantities will be averaged). For example, there are 416 on-peak hours and 328 off-peak hours (for a total of 744 hours) in July 2010. The 416 on-peak hourly supply curves are averaged and used to construct the average on-peak reference supply curve, and the 328 off-peak hourly supply curves are averaged and used to construct the average off-peak reference supply curve. The on-peak and off-peak reference supply curves are illustrated in Figure 2.

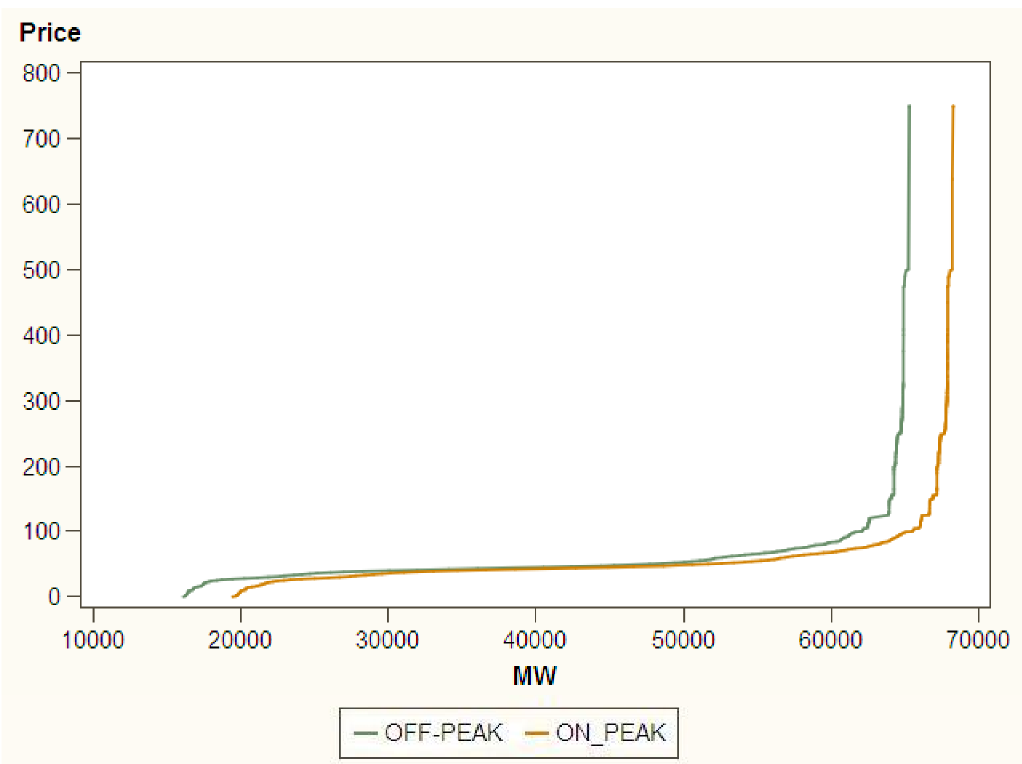

FIGURE 2: A SAMPLE SUPPLY CURVE FROM JULY 2010

[1] NERC, http://www.nerc.com/docs/oc/rs/Additional_Off-peak_Days.doc

FERC order 745 requires the reference supply curve be adjusted for fuel price differences between the reference month and the trade month. Gas fired units account for approximately 60% of the installed capacity in the ISO, while oil units and coal units each account for 1%. Because the oil and coal percentages are so small relative to gas, the ISO will only adjust for gas price differences in the Net Benefits Test. The ISO intends to use the simple average of the following two indices to calculate the California gas price:[2]

- PG&E Citygate, and
- Southern California Border

The supply curve will be scaled by a scaling factor, which is defined as the forward gas price for the trade month divided by the historical average gas price for the reference month. More specifically, for every supply quantity, the corresponding bid price will be scaled by the scaling factor. For example, if the forward monthly average gas price is $4.73 for July 2011,[3] and the historical monthly average gas price was $4.25 for July 2010, then the gas scalar = 4.73/4.25 = 1.11.

Scaling the supply curve factors in both the fuel cost difference for gas fired units and the opportunity cost differences for generators of other fuel types. Even though the whole supply curve is scaled, only the portion that is close to the threshold price is relevant for calculation of the threshold. With typical threshold prices around $45 to $60, the supply bids in this range are mainly from gas fired units or generators of other fuel types whose bids incorporate opportunity costs. Therefore, it is appropriate to scale the system wide supply curve without needing to drill down to the unit specific level.

In summary, for each trade month, the ISO will have an on-peak representative supply curve and an off-peak representative supply curve, which accounts for resource availability and fuel price differences between the reference month and the trade month.

2.2 CURVE SMOOTHING

FERC order 745 requires the supply curve be smoothed using numerical methods. The curve will be smoothed to twice differentiable so that theorem 1 and theorem 2 can be used to characterize the threshold point.

The smoothing method proposed by the ISO is an exponential function curve fitting expressed as

p = exp(*a**q^3+*b**q^2+*c**q+*d*),[4]

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[2] The ISO is working on acquiring reliable data source for these two gas price indices. However, if the data source is unavailable, the ISO will use the Henry Hub price index instead.

[3] The $4.73 forward gas price is only intended to demonstrate how to calculate the gas scalar, and may not be the actual monthly average forward gas price.

[4] Midwest ISO adopts similar function form, https://www.midwestiso.org/Library/Repository/Meeting%20Material/Stakeholder/DRWG/2011/20110509/20110509%20DRWG%20Item%2003b%20Net%20Benefit%20Test%20for%20Demand%20Response%20Compensation.pdf

where a, b, c, and d are coefficients to be determined by a regression on observations of supply quantities and prices.

The regression can be carried out by taking the natural logarithm of the price:

$\ln(p) = a*q^3+b*q^2+c*q+d$.

This converts the regression from non-linear to linear.

One technique to achieve a better fit is to apply a price window to the representative supply curves such that the threshold price is inside the price window. In this way, observations that are far away from the threshold, which are irrelevant for the Net Benefits Test, will not affect the regression. In other words, a properly chosen price window allows the regression to focus on observations that are close to the threshold in order to more accurately estimate the threshold price point. On the other hand, the price window should not be too small. If the threshold is too small, it is possible that the threshold price resides outside this price window. If this happens, the price window must be adjusted, and the regression process repeated until the threshold price is well situated inside the price window. Choosing a window from $25 to $100 produces good results from the historical data. Sample smoothed supply curves for July 2011 are illustrated in Figure 3 and Figure 4. In this example, the parameters of the smoothed curves are listed in

Table 1.

Coefficients	Off-peak	On-peak
a (*10^(−9))	0.00004274	0.0000465
b (*10^(−6))	-0.0049986	-0.0059874
c (*10^(−3))	0.20570776	0.2678375
d	0.96260595	-0.2399994

TABLE 1: SAMPLE JULY 2011 REGRESSION RESULTS

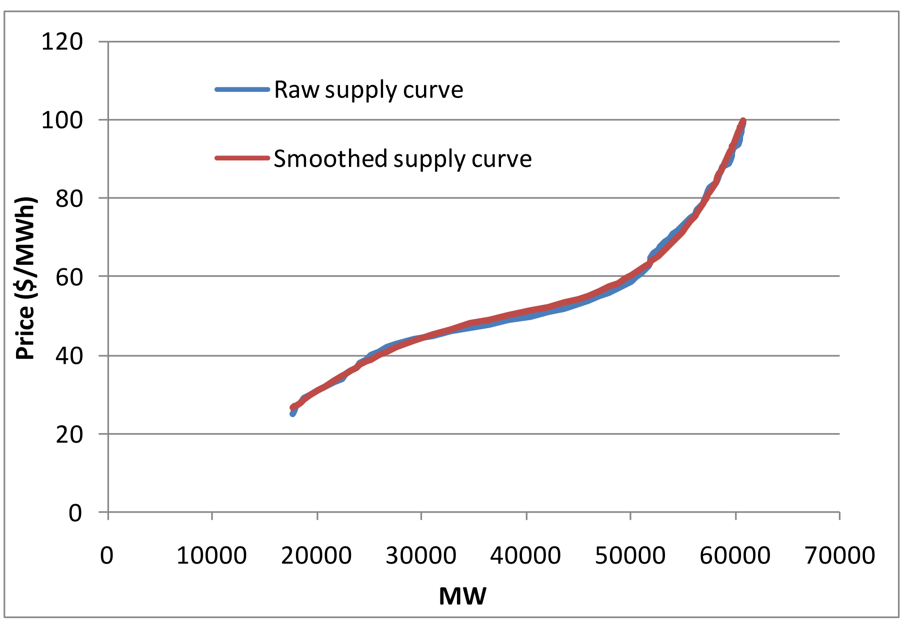

FIGURE 3: SMOOTHED OFF-PEAK SUPPLY CURVE FOR JULY 2011 WITH PRICE WINDOW [25, 100]

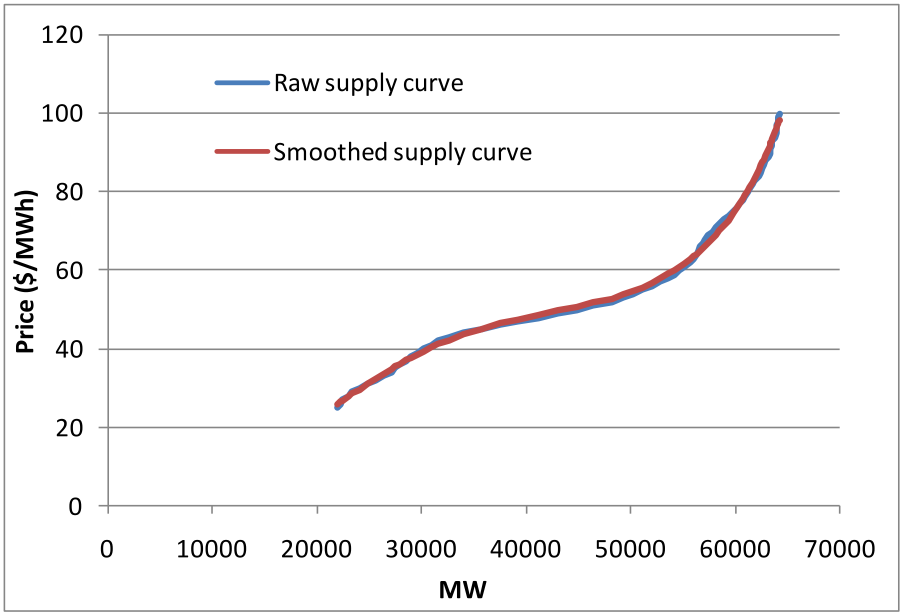

FIGURE 4: SMOOTHED ON-PEAK SUPPLY CURVE FOR JULY 2011 WITH PRICE WINDOW [25, 100]

2.3 FINDING THE THRESHOLD PRICE

Given the supply curve in the form of p = exp(*a**q^3+*b**q^2+*c**q+*d*), the threshold price is first calculated using the first order necessary condition (the elasticity equal to one) as follows:

(dp/dq) /(p/q) = 1, or

(3**a**q^2+2**b**q+*c*)* exp(*a**q^3+*b**q^2+*c**q+*d*) / [exp(*a**q^3+*b**q^2+*c**q+*d*) / q] = 1, or

3**a**q^3+2**b**q^2+*c**q =1.

Solve this cubic equation, and denote the root by q0.

This is a cubic equation, so there are three roots. If there is one real root, and two complex roots, then the real root should be used to calculate the threshold price. If there are three real roots, then:

- The one produces a price outside the price window should be discarded.
- The one, at which the supply curve is concave, should be discarded by theorem 1.

In the July 2011 on-peak example, the three roots are 4646.7, 30329.4, and 50864.8, and the corresponding prices are $2.41, $39.37, and $55.26. The price $2.41 is outside the price window, so it should be discarded. At the price $39.37, the supply curve is concave, so it should also be discarded. The price of $55.26 is the only point that satisfies theorem 1. In addition, because the supply curve is strictly convex at the price of $55.26, it is a true threshold price locally per theorem 2. Similarly, the true threshold price for July 2011 off-peak hours is $57.00.

3. RESULTS

Preliminary results based on actual historical market bids without gas price adjustment typically produce threshold prices of $45 to $60.

APPENDIX

Theorem 1 [second order necessary condition]: Assuming the supply curve is monotonically increasing and twice differentiable, if there exists a point (q0, p0) on the supply curve with q0 > 0 and p0 > 0 that satisfies the first order necessary condition (the supply curve has elasticity equal to one at q0), and for all p > p0, dp/dq >= p/q, then the supply curve is convex at q0, i.e.

d^2p/dq^2(@q0) >= 0.

Proof:

Suppose (q0, p0) is a point satisfies the first order necessary condition, [dp/dq(@q0)] / (p0/q0) = 1, and for all p > p0, dp/dq >= p/q.

By first order Taylor expansion, dp/dq = dp/dq(@q0) + [d^2p/dq^2(@q0)] * (q-q0).

By first order Taylor expansion, p/q = p0/q0 + [(dp/dq*q – p) / q^2](@q0) * (q-q0) = p0/q0.

Then, dp/dq >= p/q implies dp/dq(@q0) + [d^2p/dq^2(@q0)] * (q-q0) >= p0/q0, or

[d^2p/dq^2(@q0)] * (q-q0) >= 0.

Because the supply function is monotonically increasing, p > p0 implies q > p0. Therefore,

d^2p/dq^2(@q0)>=0.

Theorem 2 [second order locally sufficient condition]: Assuming the supply curve is monotonically increasing and twice differentiable, if the following conditions hold at a point (q0, p0) with q0 > 0 and p0 > 0 on the supply curve:

2A) the supply curve has elasticity equal to one at q0, i.e. [dp/dq(@q0)] / (p0/q0) = 1, and

2B) the supply curve is convex at q0, i.e. d^2p/dq^2(@q0) > 0,

then for all p > p0 in the vicinity of p0, dp/dq > p/q.

Proof:

Similar as the proof of Theorem 1,

d^2p/dq^2(@q0) > 0 implies [d^2p/dq^2(@q0)] * (q-q0) > 0 for all p > p0 in the vicinity of p0.

Because [dp/dq(@q0)] / (p0/q0) = 1, dp/dq(@q0) = p0/q0.

Therefore, dp/dq(@q0) + [d^2p/dq^2(@q0)] * (q-q0) > p0/q0.

By first order Taylor expansion of dp/dq and p/q, dp/dq > p/q for all q > q0 in the vicinity of q0.

Because the supply curve is monotonically increasing, dp/dq > p/q for all p > p0 in the vicinity of p0.